



Observatoire
de la CÔTE d'AZUR



Université
Nice SOPHIA ANTIPOLIS



INSU
Observer & comprendre

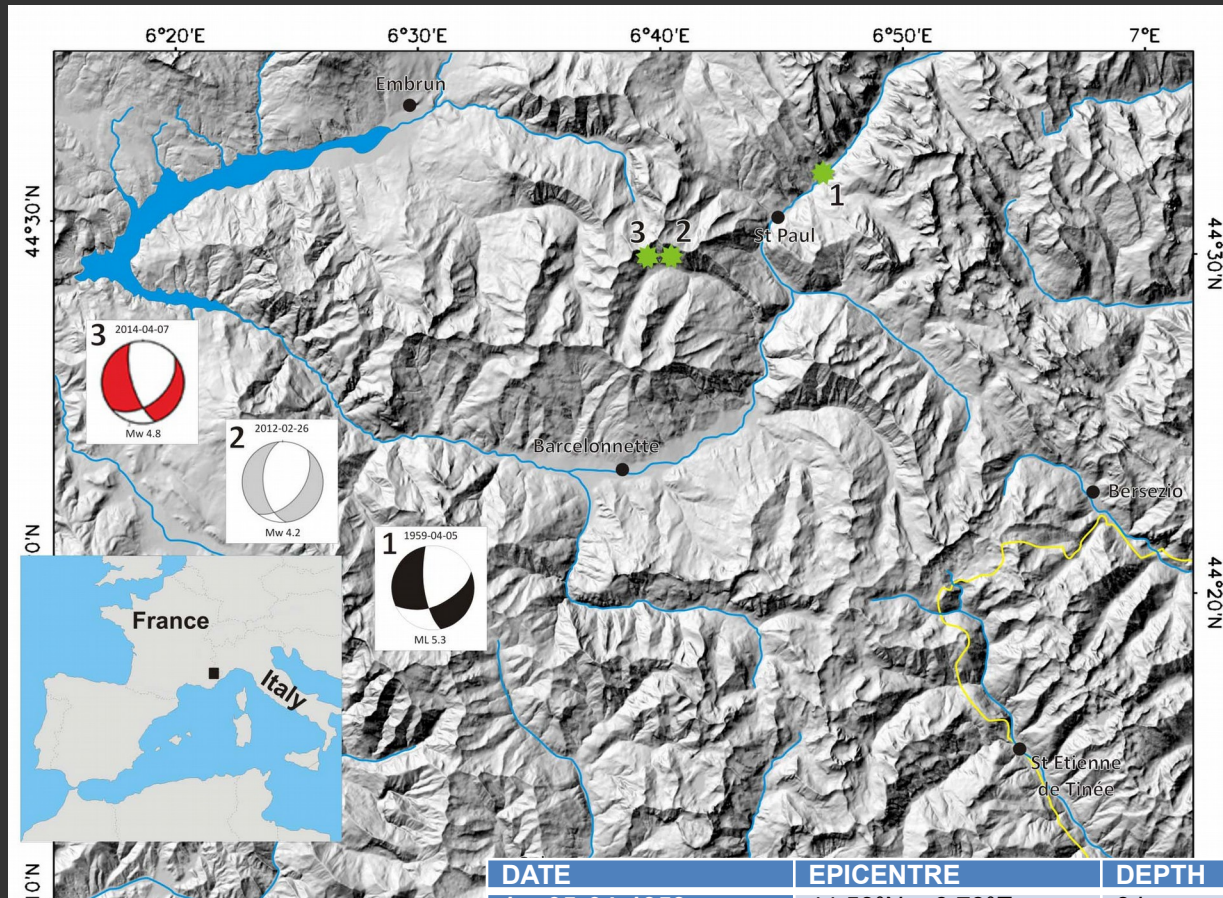


Crash course on Kinematic inversion



A. Sladen – CNRS, Géoazur

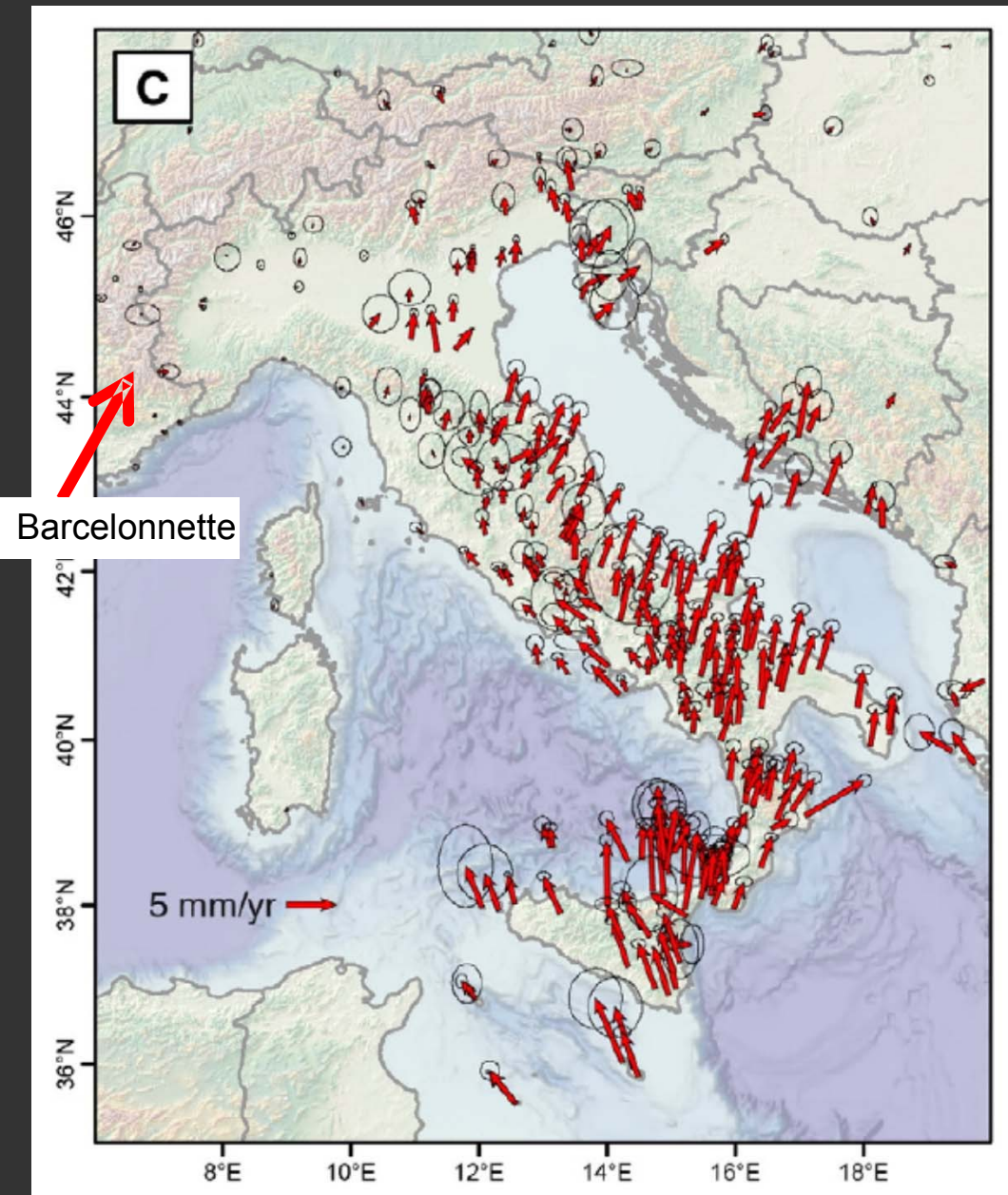
Barcelonnette



DATE	EPICENTRE	DEPTH	MAGNITUDE
1- 05-04-1959	44,53°N – 6,78°E	8 km	ML 5,3
3- 26-02-2012	44,49°N – 6,66°E	9,2 km u.s.l.	Mw 4,2
2- 07-04-2014	44,49°N - 6,68°E	11,2 km u.s.l.	Mw 4,8



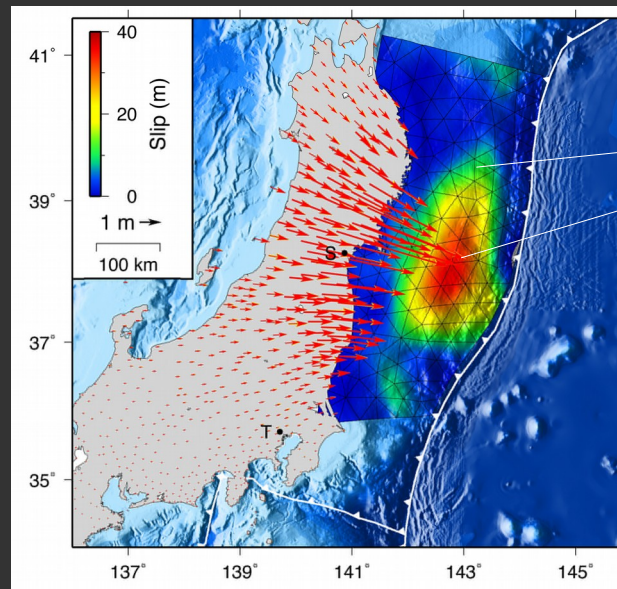
Barcelonnette



Not tectonic
High pressure fluid migration?
Erosion?
Thickening of crust?

A practical for next joint
inversion school?

Kinematic Finite fault source inversion



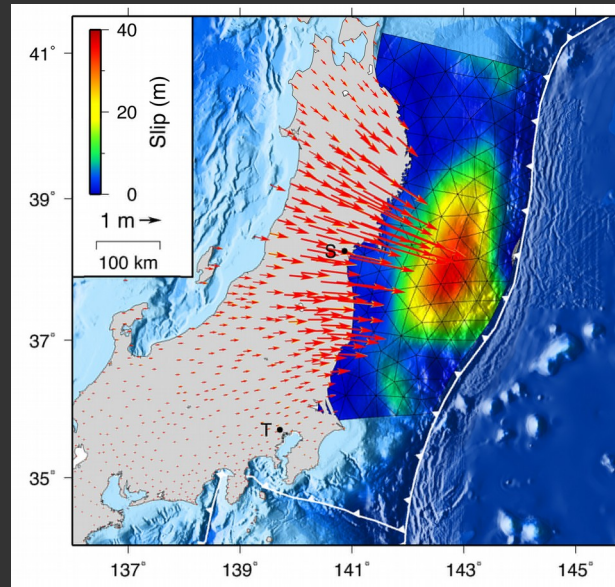
At what time?
for how long?

{ Inversion
Tomography
Imagery }

{ Slip
Source
Rupture }

{ Static
Kinematic
~~Dynamic~~ }

Kinematic Finite fault source inversion



KINEMATIC PBL

STATIC PBL

+ slip orientation
+ slip amplitude

+ rupture speed V_r
+ slip duration T_r

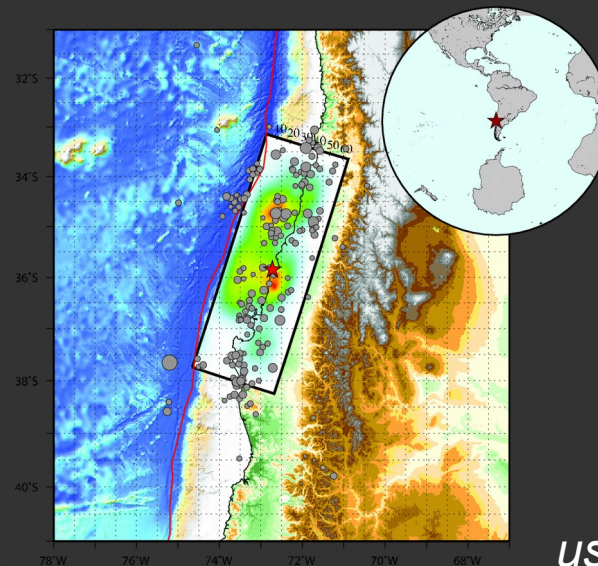
{ Inversion
Tomography
Imagery }

{ Slip
Source
Rupture }

{ Static
Kinematic
~~Dynamic~~ }

Why you might be interested?

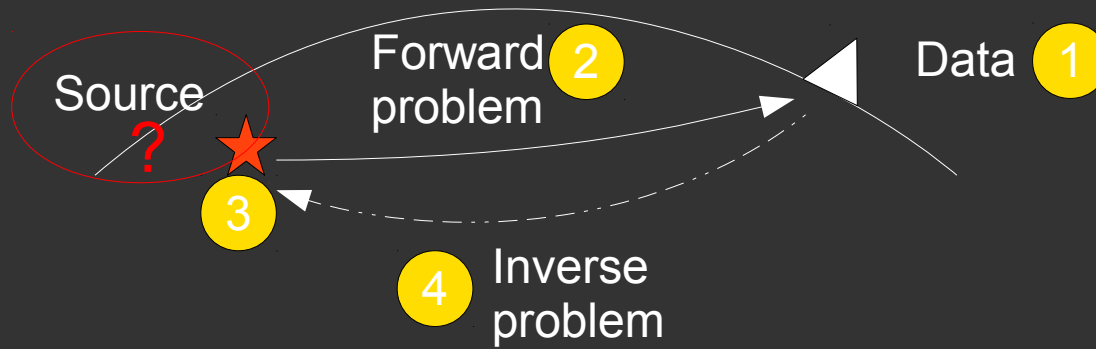
- want to do kinematic inversion (I include steps and links to open codes)
- treating a problem with time evolution
- need “slip models” for your research
- curious



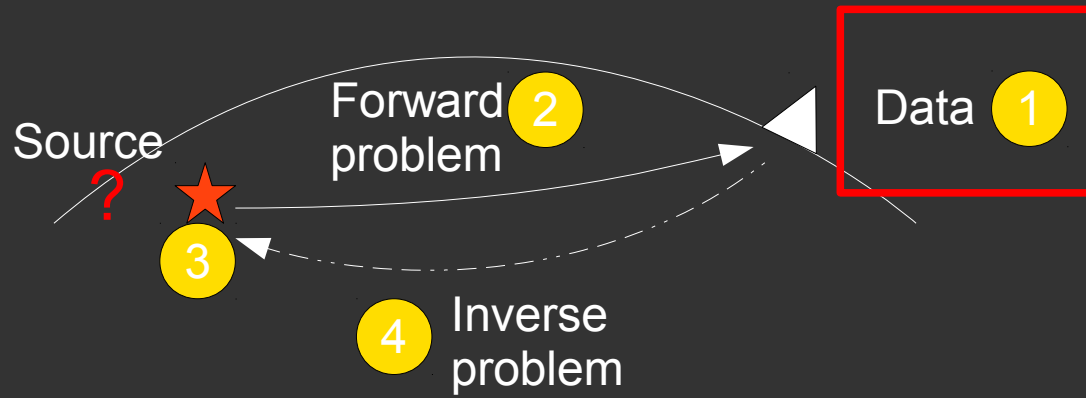
Kinematic Finite fault source inversion : why ?

- Do ruptures have **generic properties** (better anticipate)? To what extent each earthquake is unique?
- Narrow down the **physics of rupture**: mechanical, chemical and/or thermal processes ?
- Estimation of **ground-motion** (most often kinematic models only simulated),
- Earthquake and tsunami **early-warning**, rapid damage assessment,
- Improve our understanding the **seismic cycle**: relation to aftershocks, postseismic slip and intersismic (coupling)
-

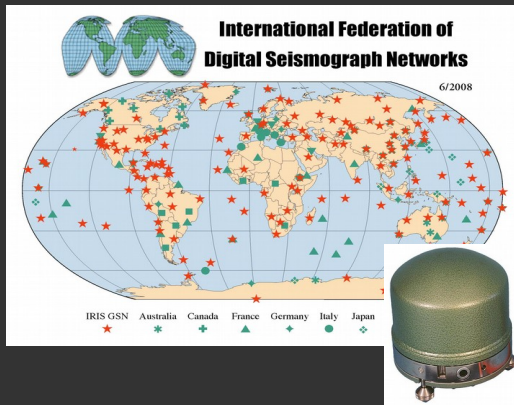
kinematic.table_of_content()



DATA

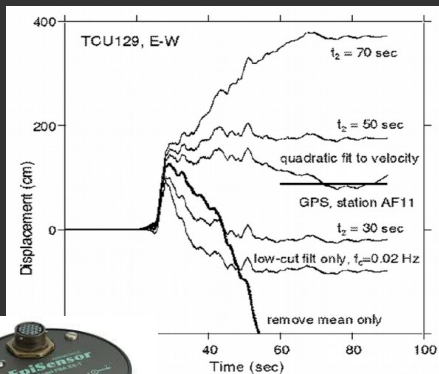
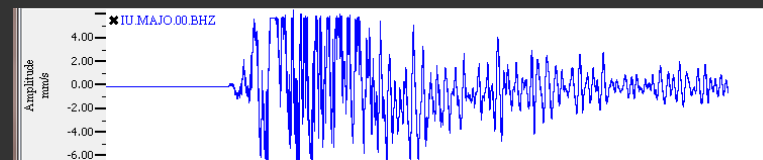


data.type()



- **Seismographs**: provide velocity, often continuous and broadband (0.01-50Hz)

But they can saturate in near-field for large events (clipped). Often use time relative to P or S-wave



D.M. Boore,
2011

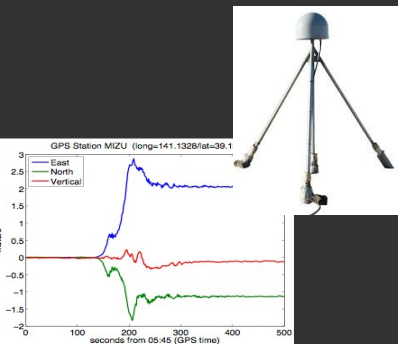


- **Accelerometers** : initially for engineering. Response function of g. No saturation, larger range of HF, absolute time (if available...)

But still many not continuous (triggered) and no precise GPS clock,

Difficult to integrate into displacement (rotation vertical axis and non-linear effects)

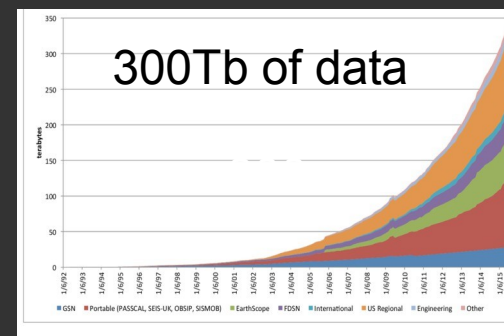
- **Continuous GPS** with high sampling rate (1 to 10sps) : directly provides displacement but complex processing, limited to nearby/shallow earthquakes in instrumented parts of the world.



data.get()

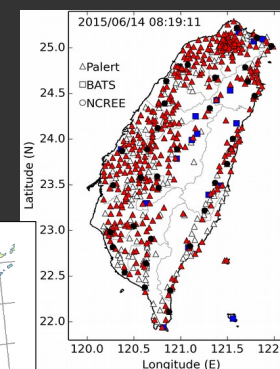
Broadband seismograms

- **www.iris.edu portal**: huge DB, 21 interactive tools (!?) and lots of by-products
- main data format: **sac** (binary+header)
- SEED (standard eq exchange data): archive with time-series, metadata, instruments responses



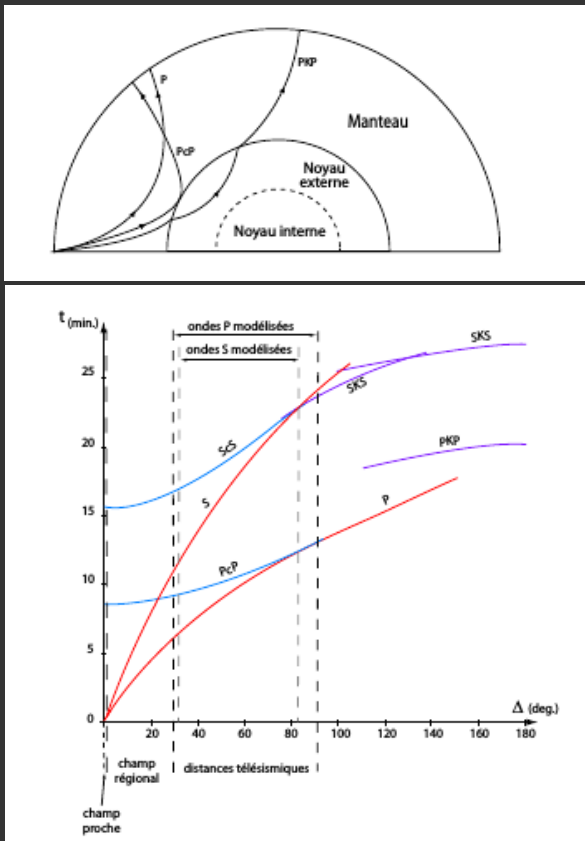
Strong-motion and continuous GPS : mainly network dependent. e.g.

- US [iris.edu](http://www.iris.edu),
- EU esm.mi.ingv.it,
- JP <http://www.kik.bosai.go.jp>



data.select()

Teleseismic bodywaves

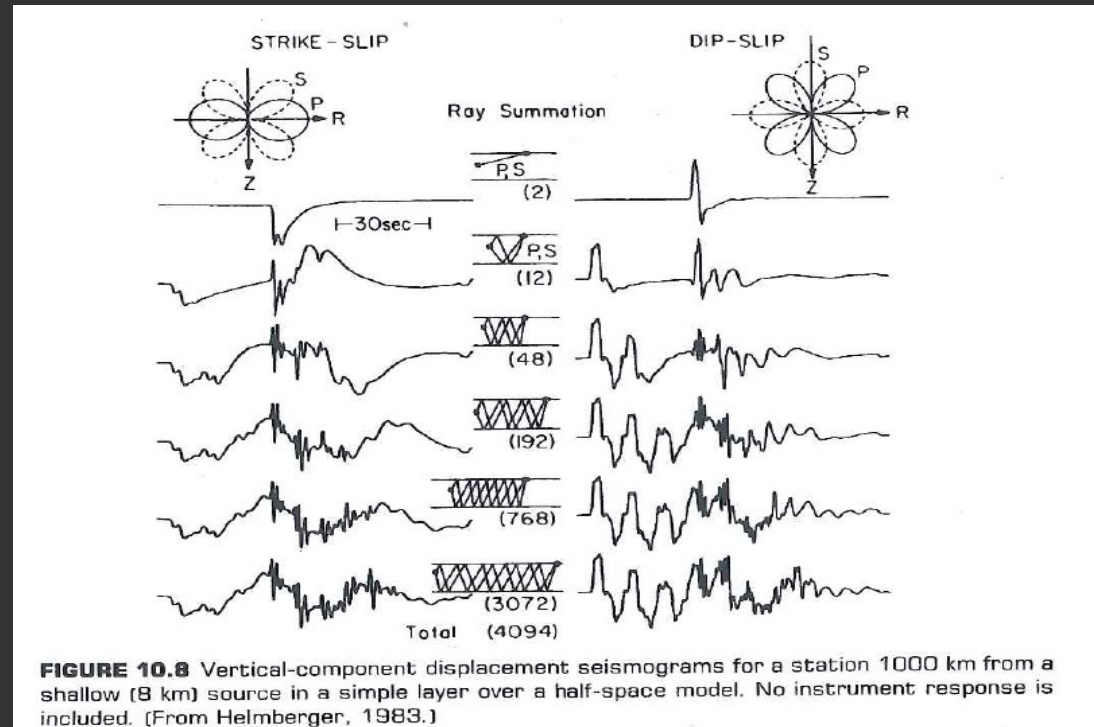


- usually focus on bodywaves because surface waves lower frequency and more complex propagation effects
- between 30° and $80-90^\circ$ of azimuthal distance: vertical take-off angle ($<20^\circ$), so little interaction with crust and upper mantle,
- typically only model P, S and related depth phases (pP, sP, pS, sS)
- we cannot model absolute travel time of teleseismic waves

data.avoid()

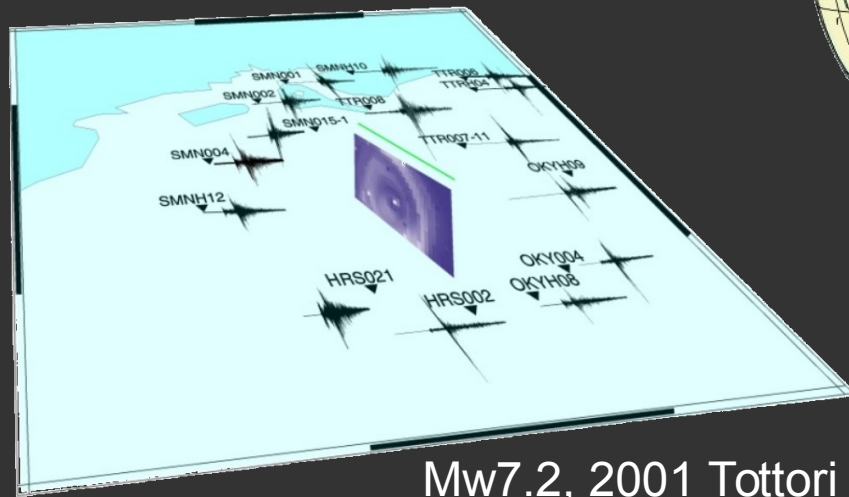
Regional data

Complex interactions in the crustal layers: at your own risk!



Lay&Wallace 1995

data.selection()



Mw7.2, 2001 Tottori



Good azimuthal coverage : limit local path effects at source and station, average noise,

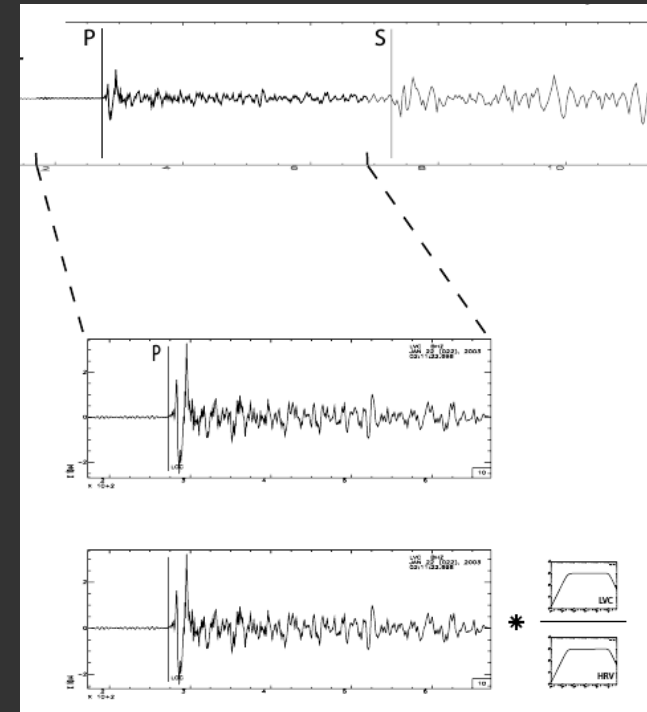
Distant (teleseismic) data not necessary if good near-field coverage (higher frequency signals with absolute timing)

data.process()

Tools

- the good ol' Seismic Analysis Code (SAC),
- the new favorite: ObsPy
-

Raw velocity
seismogram

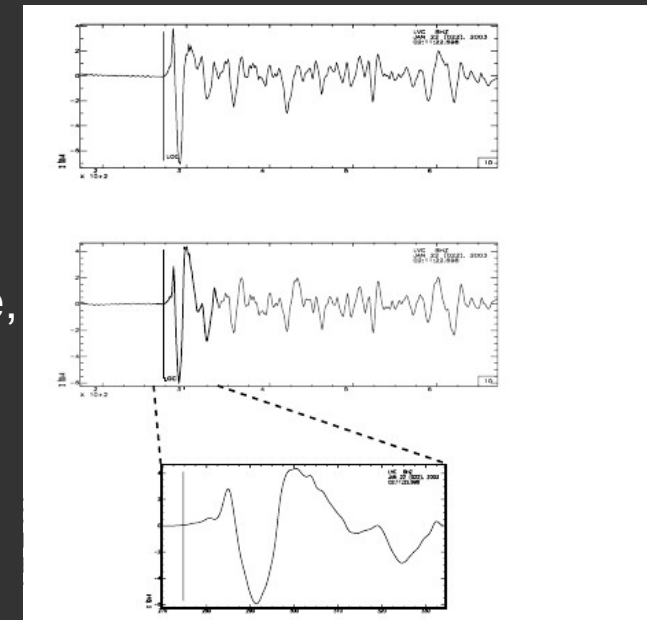


Pick P, cut,
remove trend
and mean

Remove
instrument
response

Integrate to
displacement

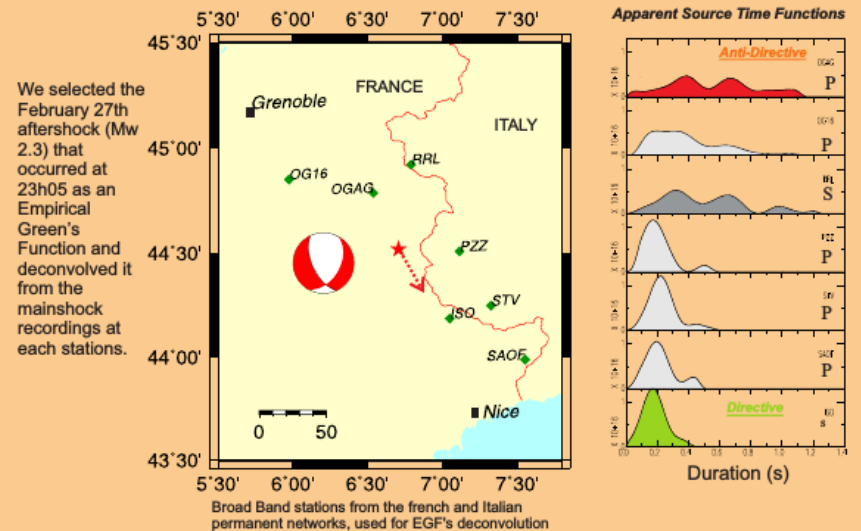
Filter, apodize,
decimate,
extract src
duration



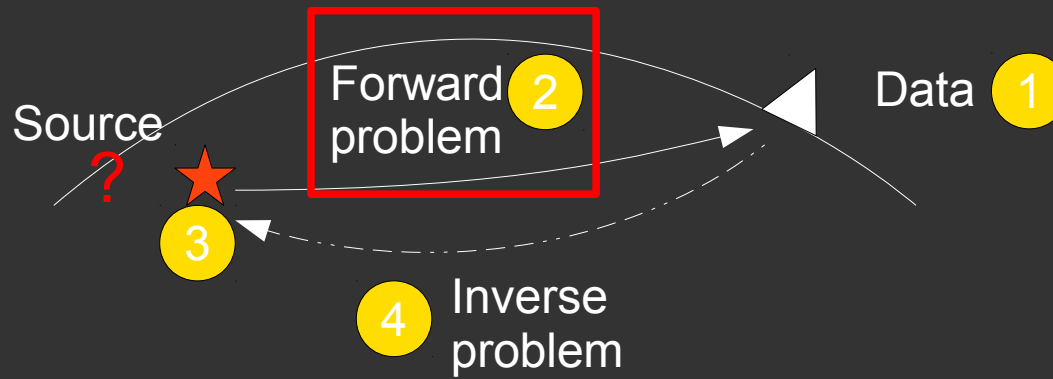
rupture propagation in seismograms



*Mw4.1,
Barcelonnette
2012*



FORWARD



forward.1d()

1D

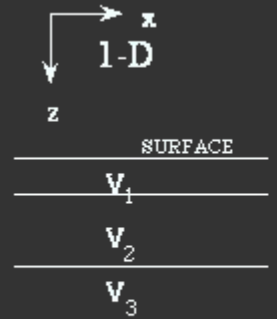
Often just as good as 3D and few cases where we know the structure at more than 0.5s (~3-4km)

Codes

- **Reflectivity**: solve equation in f-k domain with continuity of displacement at interfaces.

- Axitra (O.Coutant, 1990),
- FK-code of (Zhu and Rivera,2002).
- COMPSYN (Spudich&Xu2003) is FD implementation: more efficient for smoothly varying medium.

→ also provide static component!



forward.1d()

Teleseismic: expand wavefield at the source with ray theory adding attenuation and geometric expansion.
→ cannot work with absolute time

Source structure (e.g. reflectivity)



Normal modes: not appropriate for short period signals like bodywaves (typically up to ~5s) but good for surface waves (typically up to ~100s)

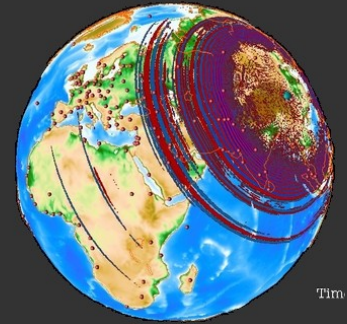
forward.3d()

3D

Finite-difference/Spectral/discrete Galerkin methods :

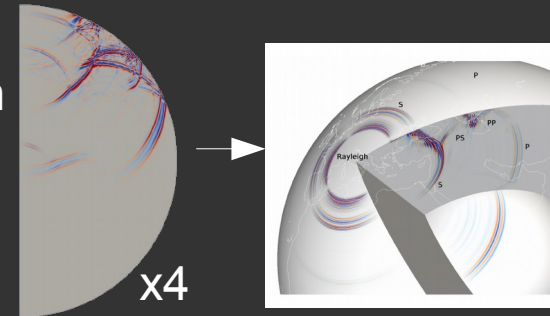
→ also provide static component depending on implementation (absorbing boundary conditions)

Too heavy computation for 3D Earth at high frequency (<10s). Mostly relevant for near-field/regional simulation (using reciprocity).



2.5D AXISEM

New spectral method : 4 simulations in 2D → 3D field in 2.5D Earth
Could be interesting in future to model phases other than P and S.
Future?



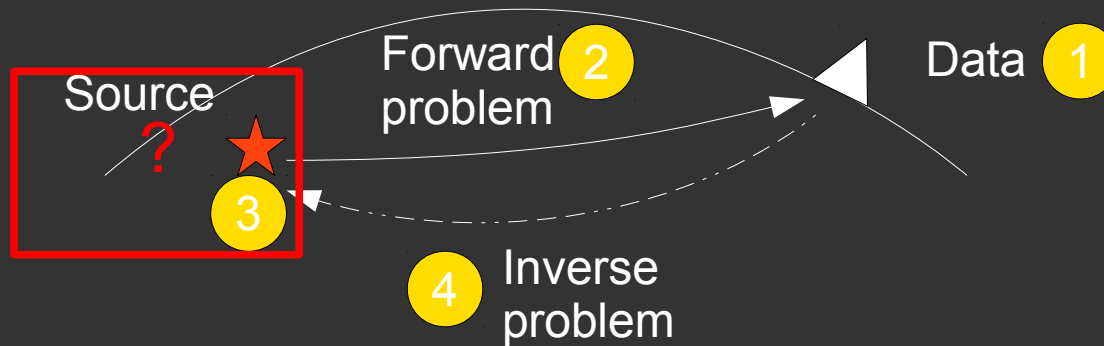
Empirical GF (S.Hartzell, GRL 1978)

→ Use smaller earthquakes as Green's functions

Requirements :

- Similar focal mechanism and location,
- $M_w \sim 2$ points smaller than mainshock,
- Ideally several EGF for the different parts of the fault

source.parameterize()

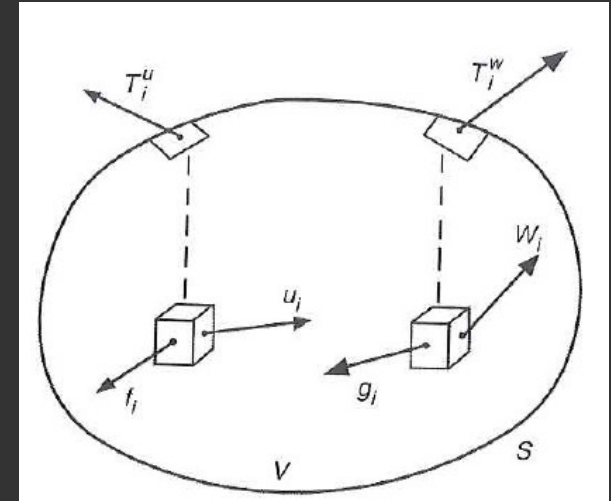


Some theoretical background

- **Reciprocity theorem**

(also known as Betti's theorem, Green-Volterra)

$$\int_{-\infty}^{\infty} dt \int_V (u_i g_i - w_i f_i) dV + \int_{-\infty}^{\infty} dt \int_S (u_i T_i^w - w_i T_i^u) dS = 0$$



→ if causality respected (ok for EQ), can describe system of forces with a equivalent, simpler one

....accelerographs

Data WWSSN
Long/short T



1960

1970

1980

1990

2000

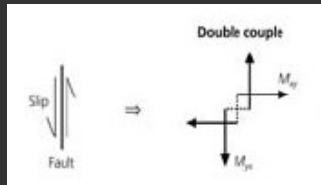
2010

Key
events

1963
Dble-
couple
model

Classic
papers

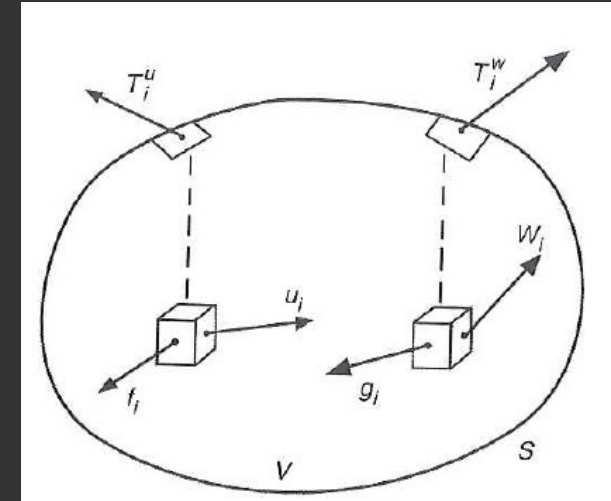
Maruyama
(1963)
Burridge &
Knopoff
(1964)



Some theoretical background

- **Reciprocity theorem** (Betti's theorem, Green-Volterra) :

$$\int_{-\infty}^{\infty} dt \int_V (u_i g_i - w_i f_i) dV + \int_{-\infty}^{\infty} dt \int_S (u_i T_i^w - w_i T_i^u) dS = 0$$



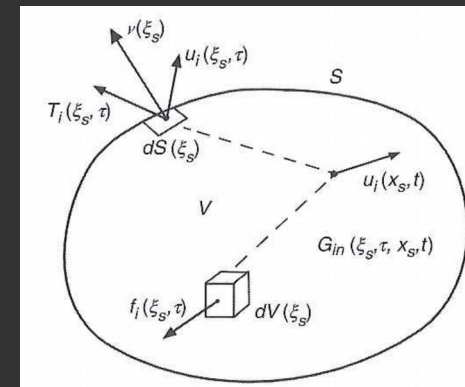
- **Representation theorem** (Volterra's theorem)

$$u_n(\mathbf{x}, t) = \int_{-\infty}^{\infty} d\tau \int \int \int_V f_i(\xi, \tau) \cdot G_{in}(\xi, t - \tau; \mathbf{x}, 0) dV + \int_{-\infty}^{\infty} d\tau \int \int_S [G_{in}(\xi, t - \tau; \mathbf{x}, 0) \cdot T_i(\mathbf{u}(\xi, \tau), \mathbf{n})] dS - \int_{-\infty}^{\infty} d\tau \int \int_S [u_i(\xi, \tau) \cdot c_{ijkl} \cdot n_j \cdot G_{kn,l}(\xi, t - \tau; \mathbf{x}, 0)] dS$$

moment
tensor
surface
density

unitary
force
couple

Surface
stress :
for
dynamic
modeling



$$u_n(\mathbf{x}, t) = M_{pq} * G_{np,q}$$

Some theoretical background

- Linear filter theory, a convenient way to model a signal, the ground motion

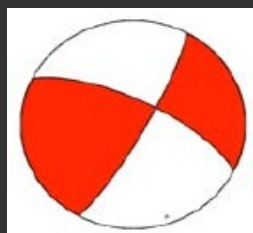
$$u_k(t) = \underline{s(t)} * \underline{g_k(t)} * \underline{i_k(t)}$$

$$\delta u(\vec{\varepsilon}, t) = A(\vec{\varepsilon}).S(t - T(\vec{\varepsilon}); \tau(\vec{\varepsilon})).r(\vec{\varepsilon})$$

$$\text{with } \int_0^\infty S(t - T(\vec{\varepsilon}); \tau(\vec{\varepsilon})) dt = 1$$

Some theoretical background

- Moment tensor (MT) inversion either:

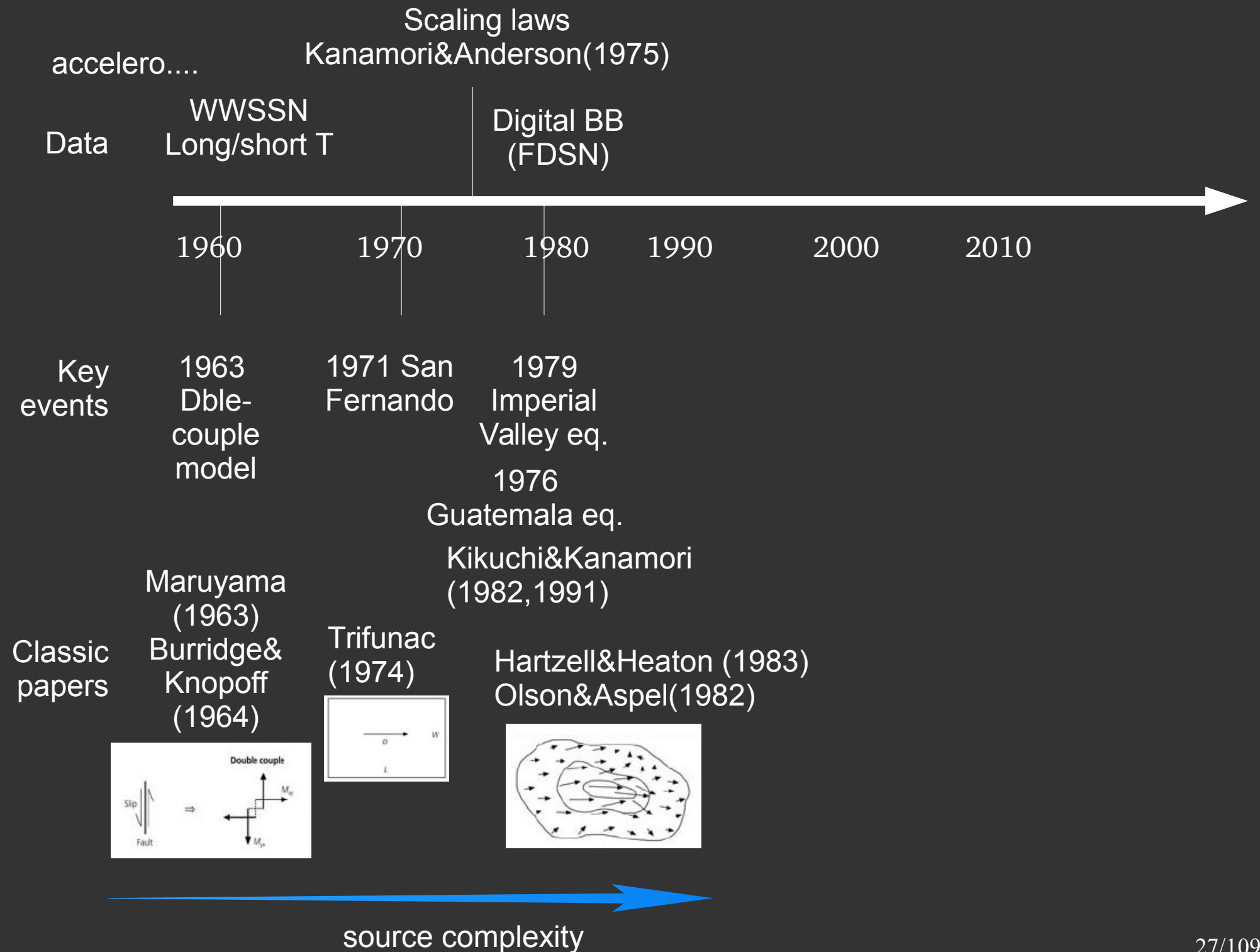


- solving the inverse problem (need damping to stabilize inversion)
- Taylor expansion of MT expression \rightarrow CMT solutions
- grid search (now common)

$$u_i(t) = \sum_{j=1}^6 G_{ij}(t) m_j$$

$$\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} & G_{26} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{n1} & G_{n2} & G_{n3} & G_{n4} & G_{n5} & G_{n6} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{pmatrix}$$

$$m^{est} = [G^T G]^{-1} G^T u$$



kinematic vocabulary

Rupture velocity

The velocity with which the rupture front propagates over the entire fault plane (i.e. a macroscopic measure); generally 70-90% of shear-wave velocity (2 - 3 km/s)

Rupture duration

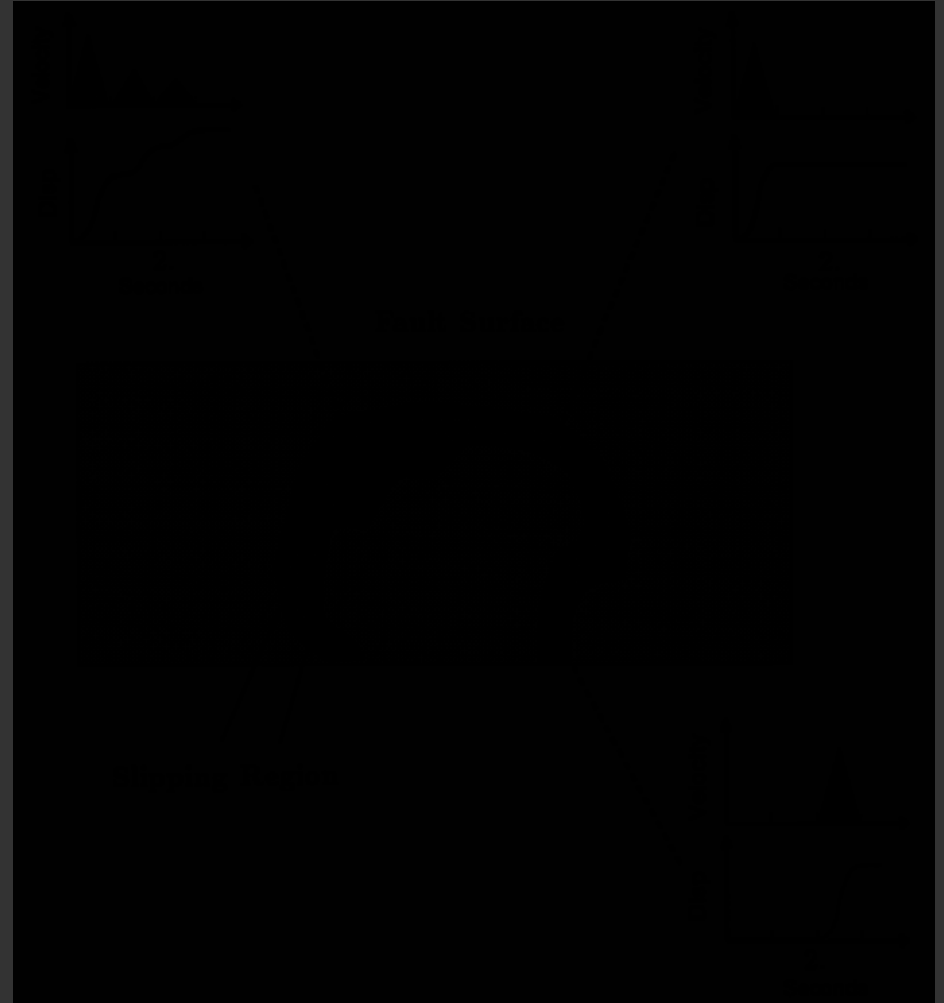
Time it takes for the earthquake to rupture the entire fault plane, i.e. from rupture nucleation until the last point on the fault stops slipping; related to rupture velocity; depends on earthquake size

Slip velocity

The velocity at which each point on the fault moves (highly variable, generally 10-100 cm/s)

Slip duration/Rise time

Length of time that each point on the fault slips; highly variable on the fault plane; strongly influences ground-motions; scales with displacement



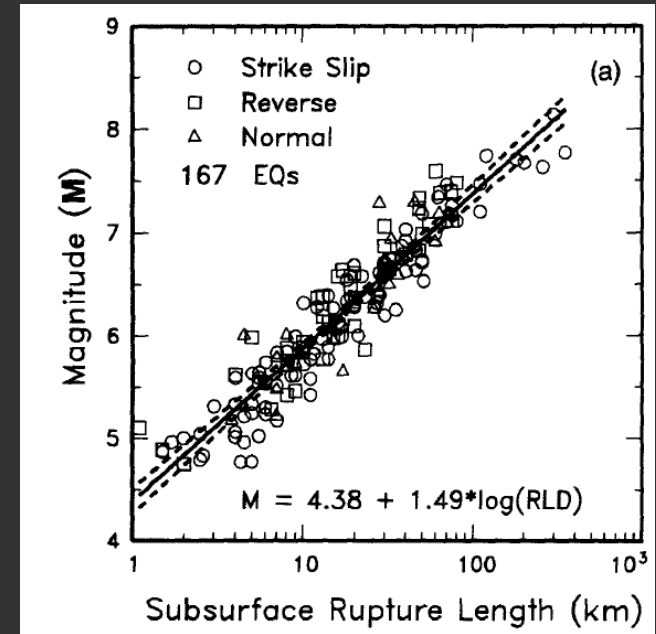
Fault geometry

→ Too non-linear to invert for fault geometry.

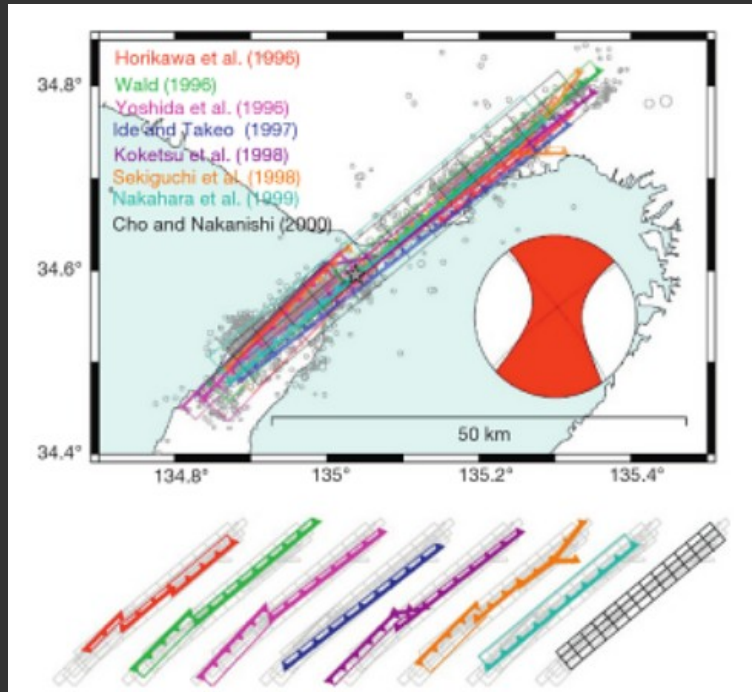
Fault geometry can be based on:

- scaling laws,
- aftershocks,
- trace of surface fault rupture,
- surface motion (e.g. upper or lower pivot line seen by geodesy)

→ difficult problem when good data

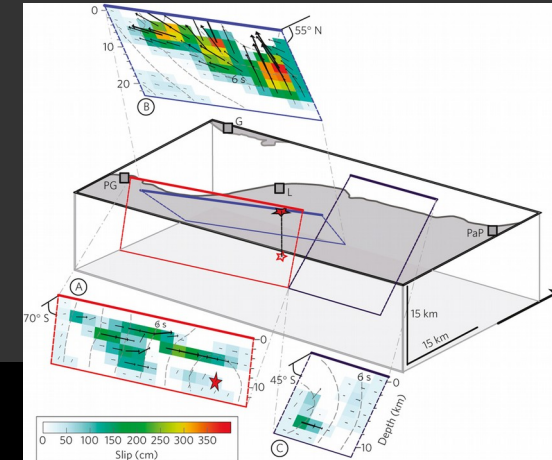


Wells&Coppersmith(1994)

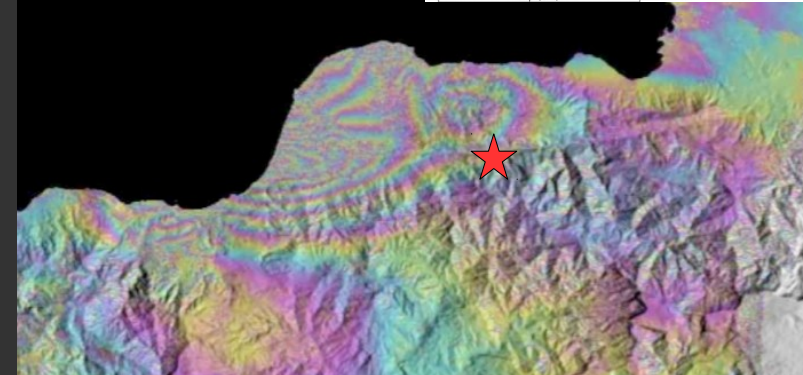


*1995 Kobe eq
(S.Ide 2007)*

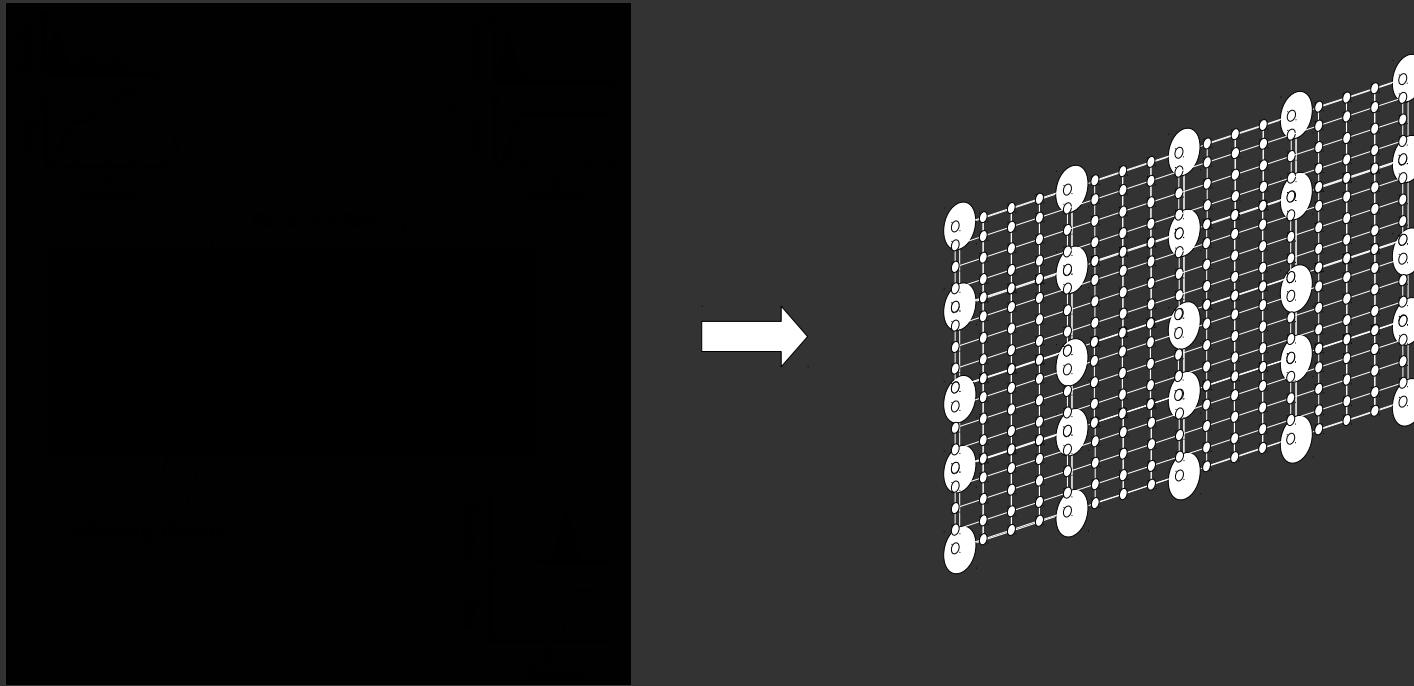
*2010 Haiti eq
(Hayes et al. 2010)*



n
sol
visee



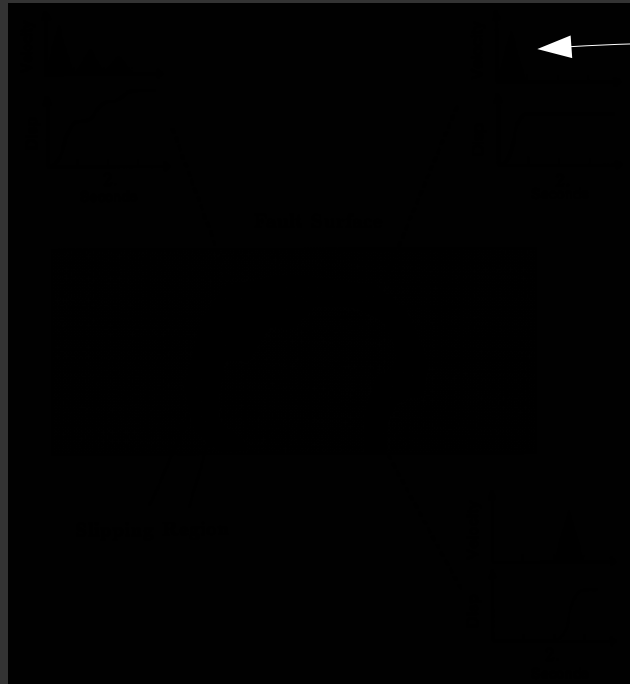
Fault parameterization in practice



Some physical constraints on the implementation

- $V_r = f(V_s \text{ or } V_p)$ depending on rupture mode. Mode-II in-plane allows supershear,
- Size grid is defined by smallest resolvable (theory): $\lambda_{min} = \frac{V_{Smin}}{f_{max}}$
- Interpolate GF at smaller scale to ensure coherence of rupture front. ~10 times more elements, especially for near-field data modeling. Can compute or interpolate with FD scheme,
- usually pre-compute all the Green's functions to save time

Fault parameterization in practice

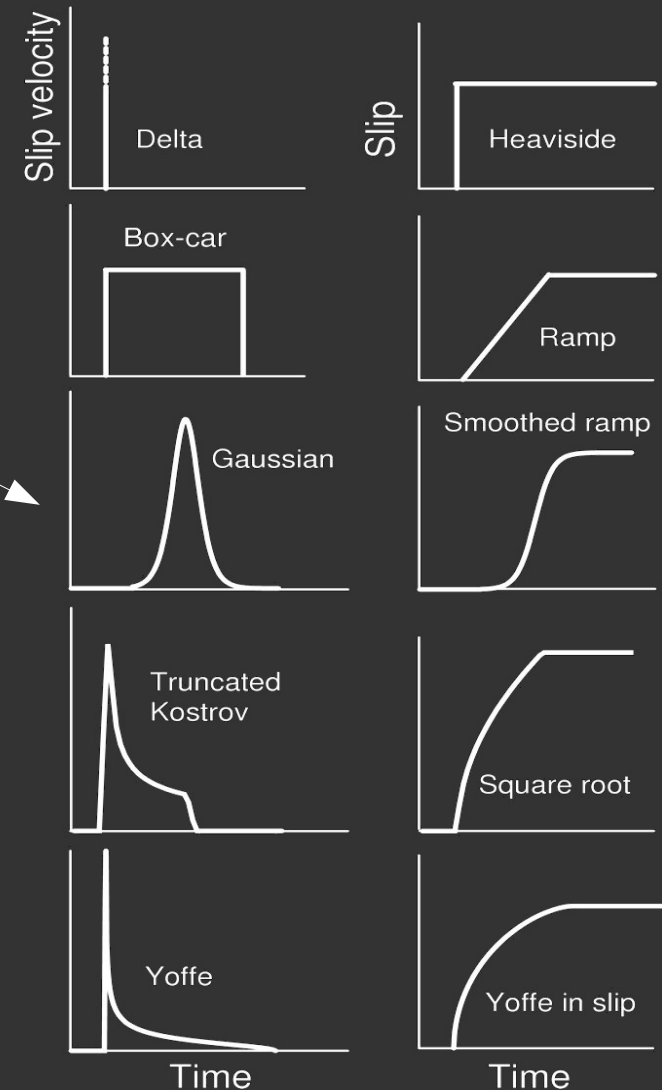


$$\delta u(\vec{\varepsilon}, t) = A(\vec{\varepsilon}).S(t - T(\vec{\varepsilon}); \tau(\vec{\varepsilon})).r(\vec{\varepsilon})$$

Slip velocity functional

- Functional form of S assumed constant to limit nb parameters but in dynamic simulations depends of acceleration of rupture (Piatanesi et al., GRL 2014)
- Observations suggest S is very short (duration of radiation < rupture duration), below 1s, so Yoffe function seems more adapted (Tinti et al., 2005). Can have influence with strong-motion data in near-field (not so far), otherwise we don't model such high freq.

Good for
slow-slip
event



kinematic.source()

How to parameterize time?

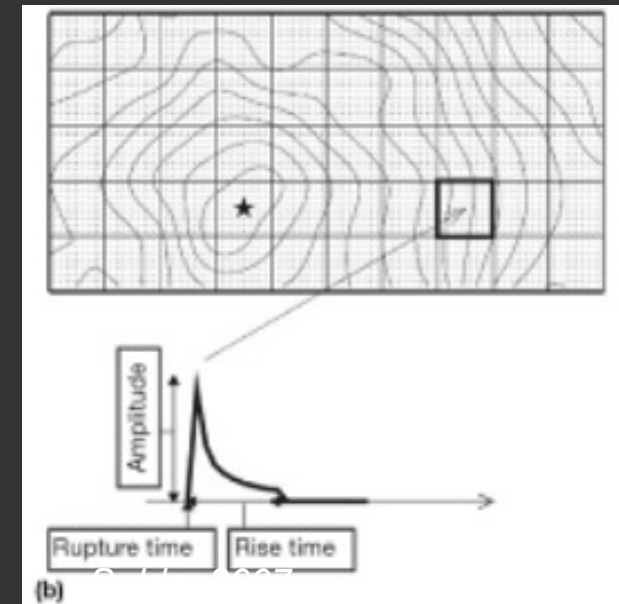
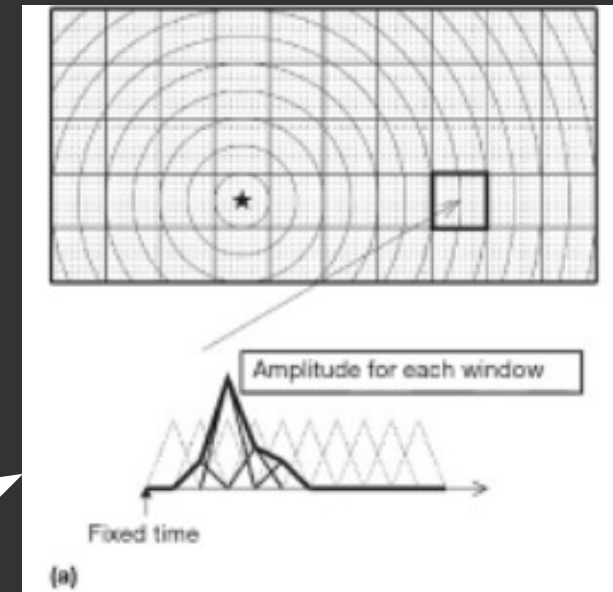
With rupture time might lead to non-causality: is V_r ideal?

- ✓ positive parameter
- ✓ can impose $V_r = f(\text{local } V_s)$.
- ✗ makes the problem non-linear

Two grand classes of approaches

- **Linear** expression with **fixed initial time**: don't use V_r and allow rupture to happen any time
- **Non-linear** expression with **variable rupture time**: treated by (1) linearizing or (2) dealing with NL
Use V_r and slip only once (Heaton « self-healing” pulse model)

Exist intermediate case where solve linear problem for slip amplitude at fixed V_r , and then NL exploration of local V_r perturbations (e.g. Fukuyama and Irikawa 1986, Takeo 1987)

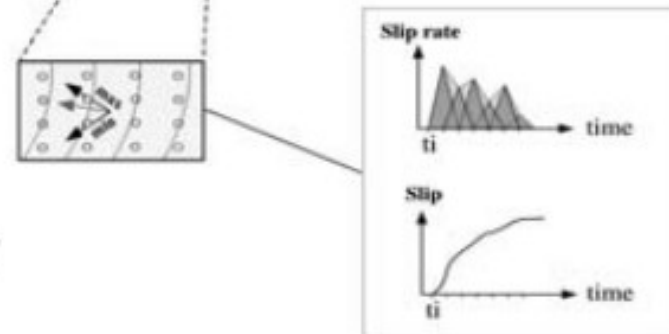
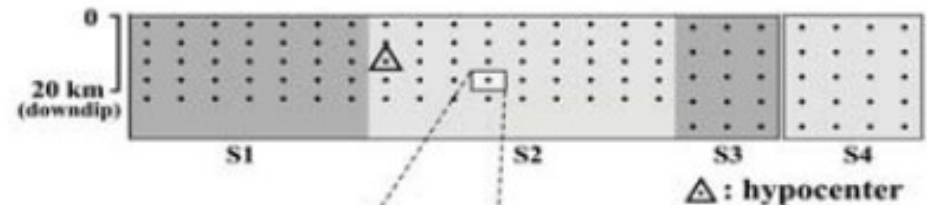


Linear « multi-time-window » approach

- Linearized inversion, using the representation theorem, by making the following assumptions (based on Olson & Apsel, 1982; Hartzell & Heaton, 1983)
 - The elementary slip function is simple and identical for all points on the fault
 - The slip-history at each point is represented by summing a number of elementary slip functions, lagged in time (multi-time window)
 - The rise-time is constant
 - The rupture speed is constant

$$\Delta t_{\text{trig}} = \frac{R}{V_r} + \Delta tw \cdot (itm - 1)$$

$$u_n(\mathbf{x}, t) = \sum_{itm=1}^{ntm} \sum_{is=1}^{ns} \sum_{if=1}^{nf} m(if, is, itm) \times \int \left[u_{\text{unit}_{is}}(\tau - \Delta t_{\text{trig}}) \right] \times c_{i(is)jkl}(\xi) n_j G_{kn,l}(\mathbf{x}, t - \tau; \xi(if), 0) d\tau$$



Linear « multi-time-window » approach

- As before, we have a linear system of equations that can be solved by common strategies

$$\underline{d} = \underline{G}\underline{m} = \sum_j G_{ij} m_j$$

include smoothing

$$\begin{bmatrix} \underline{d} \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{G} \\ \lambda \underline{S} \end{bmatrix} \underline{m}$$

S represents a smoothing matrix that accounts for variations in the model parameters with distance and time (the farther apart subfaults are, the larger a difference is allowed); λ has to be determined by trial-and-error, or some statistical information criterion.

$$\begin{array}{c} \text{(time) data points station 1} \\ \vdots \\ \text{(time) data points station 2} \\ \vdots \\ \text{(time) data points station N} \end{array} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_1 \\ d_2 \\ \vdots \\ d_1 \\ d_2 \\ \vdots \end{bmatrix} \approx \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1m} \\ G_{21} & G_{22} & \cdots & G_{2m} \\ \vdots & \vdots & & \vdots \\ G_{11} & G_{12} & \cdots & G_{1m} \\ G_{21} & G_{22} & \cdots & G_{2m} \\ \vdots & \vdots & & \vdots \\ G_{11} & G_{12} & \cdots & G_{1m} \\ G_{21} & G_{22} & \cdots & G_{2m} \\ \vdots & \vdots & & \vdots \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ \vdots \\ s_m \end{bmatrix}$$

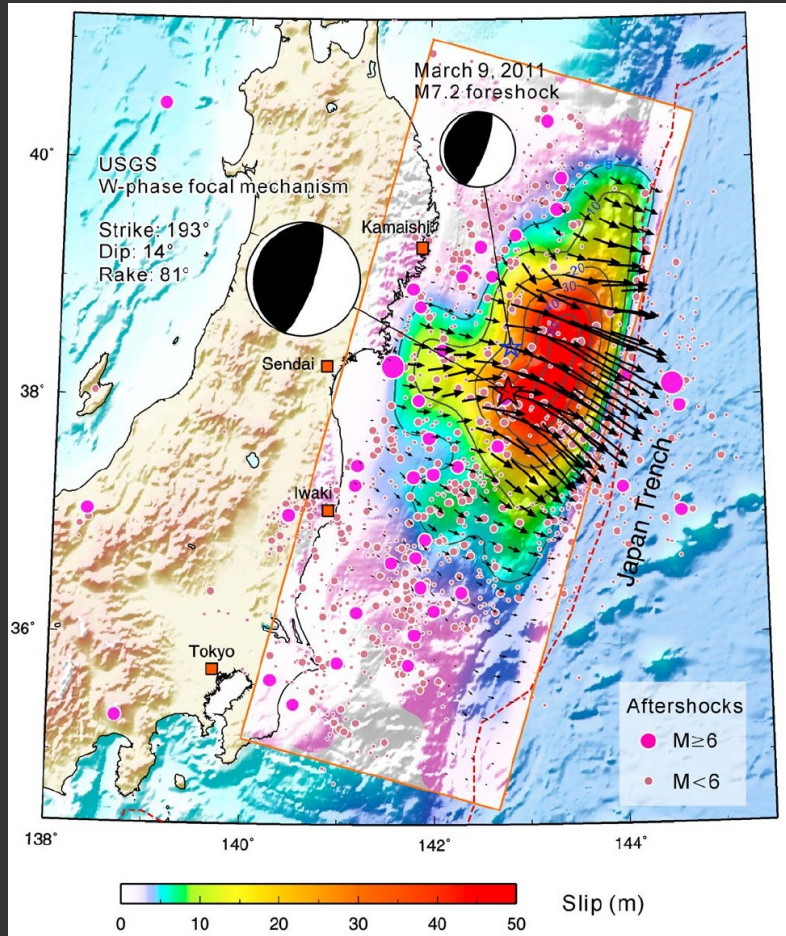
Dislocation in subfault 1
Dislocation in subfault 2

Dislocation in subfault m

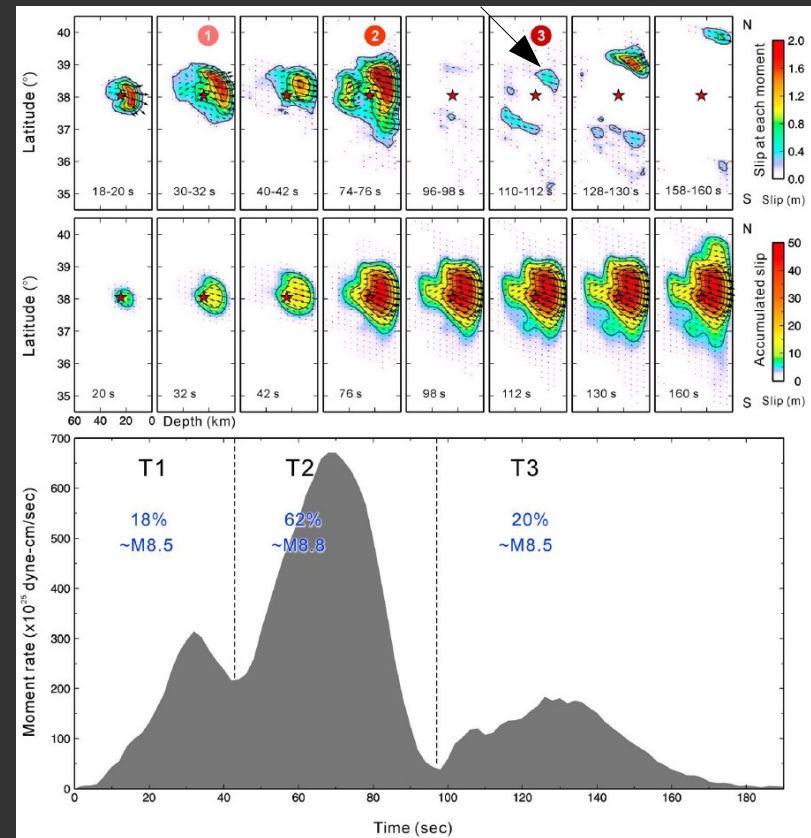
Subfault 1 synthetics Subfault 2 synthetics Subfault m synthetics

Exemple

“Evidence of large scale repeating slip during the 2011 Tohoku-Oki earthquake”
Big implications on rupture mechanics

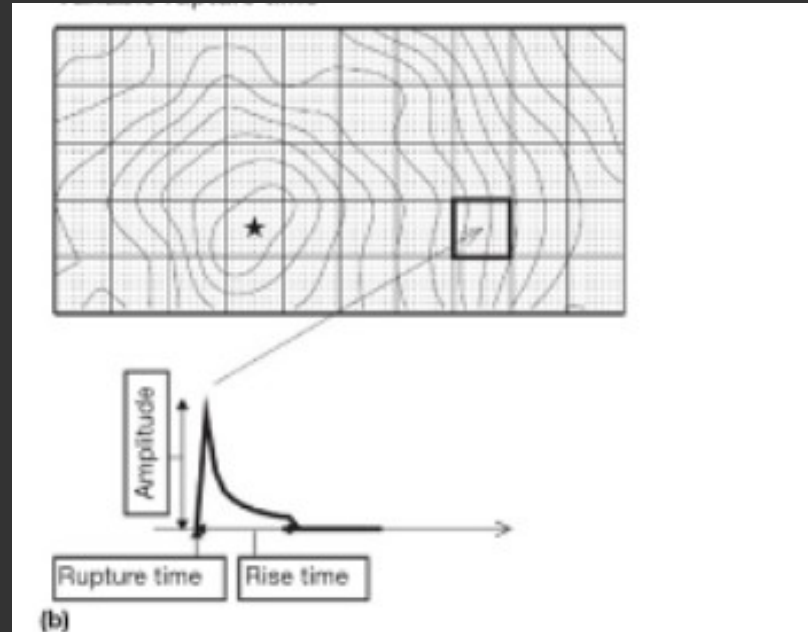


Repeating slip?



Lee et al. (2011)

Non-linear « single time window » approach



S. Ide, 2007

- 1980th: Linearize with something similar to non-linear least square solution of Tarantola and Valette (1982).
 - Assume an initial model, which will have high impact on the final solution,
- starting 1990th: bigger computer → use stochastic (Monte-Carlo) optimization methods like N.A, S.A, G.A... no initial guess needed, but hyper-parameters to control exploration. Can speed-up convergence with linearized approach.

src.parameterize()

	Multi-time-window	Single-time-window	
Typical solving algo.	NNLS (non-negative least-squares)	iterative, non-linear least-sq	Stochastic Monte-Carlo methods (SA, GA, NA...)
Computation/convergence	fast	fast	slow (can finish cvg with linearized approach)
Extra subjective parameter	no	yes (initial guess)	yes (algo cvg param.)
Slip fonction	high flexibility	can use explicit form, but often use overlapping triangles	can use explicit form, but often use overlapping triangles
Resolve Vr	Implicit	Explicit	Explicit
Repeating slip	forced	optional	optional
Regularization (nb parameters)	High (O3)	Intermediate (O2)	Intermediate (O2)
Performance on synthetics tests	introduce artificial complexity w/o fit improvement. Mo not well recovered (Cohee&Beroza94)	Good simpler solutions with almost as good fit. Mo better recovered.	

The « Iterative deconvolution » approach

- Popular method developed by Kikuchi and Kanamori (1982 ;1991;1993).

Assume that earthquake results from rupture discrete asperities : iteratively deconvolve seismograms looking for high_slip_patches(amplitude, t), assuming no causality between subevents.

Mostly appropriate to low-frequency analysis, the analysis of large earthquake at teleseismic distance

Line source

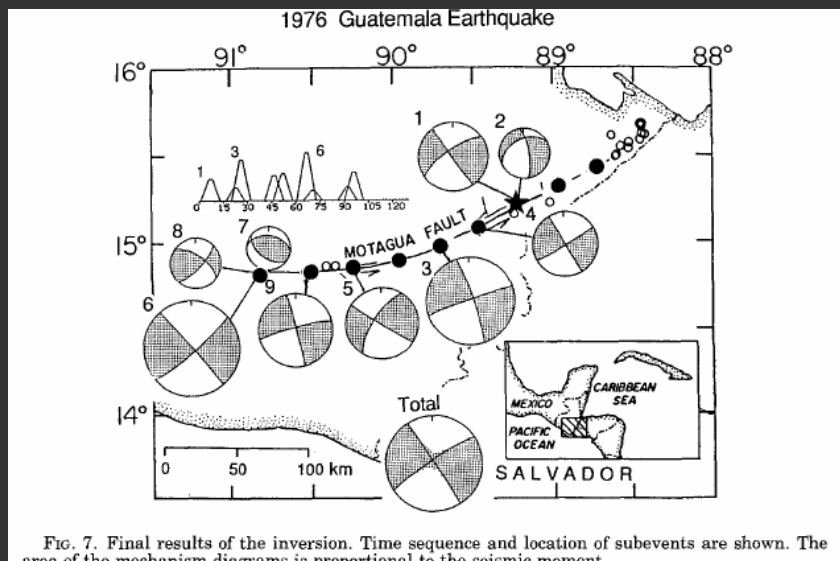
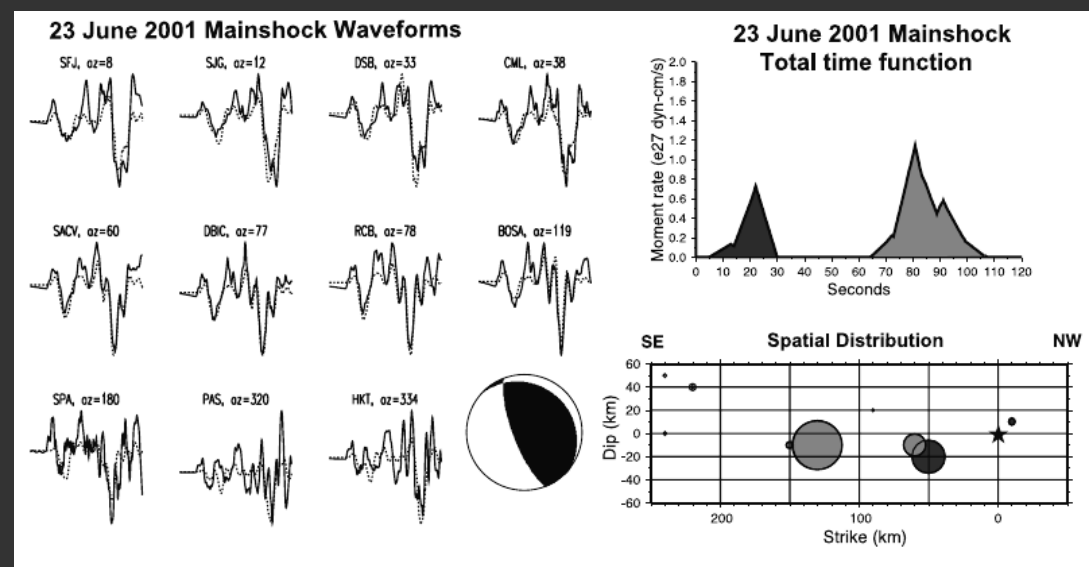


FIG. 7. Final results of the inversion. Time sequence and location of subevents are shown. The area of the mechanism diagrams is proportional to the seismic moment.

Kikuchi & Kanamori, BSSA, 1991

Later extended to finite-fault



Giovanni et al., GRL 2002

Kikuchi, M., & Kanamori, H., Bull. Seism. Soc. Am., 72, 491-506, 1982.

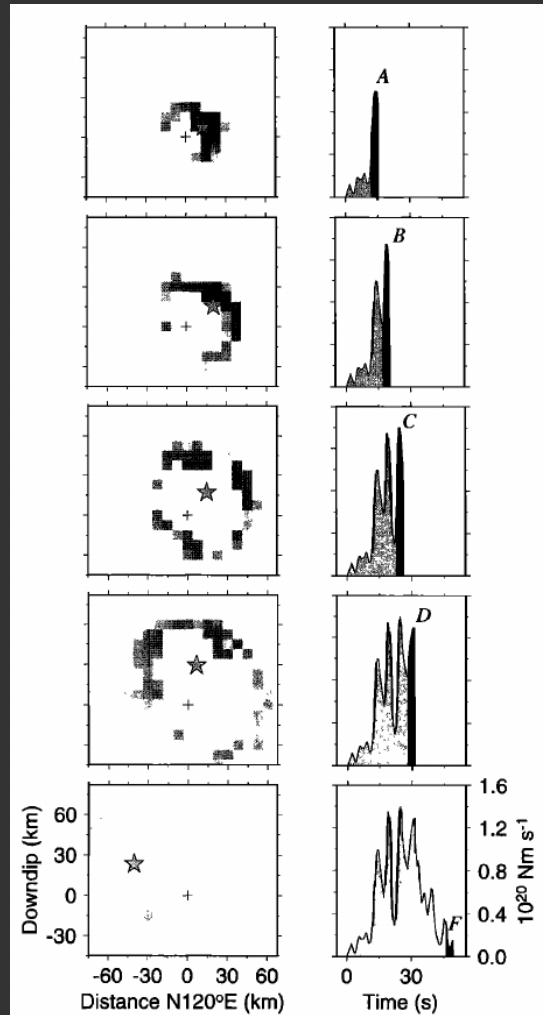
Kikuchi, M., & Kanamori, H., Bull. Seism. Soc. Am., 81, 2335-2350, 1991.

Kikuchi, M., Kanamori, H. & Satake, K., J. Geophys. Res., 98, 15797-15808, 1993.

code available @ <http://www.eic.eri.u-tokyo.ac.jp/ETAL/KIKUCHI/>

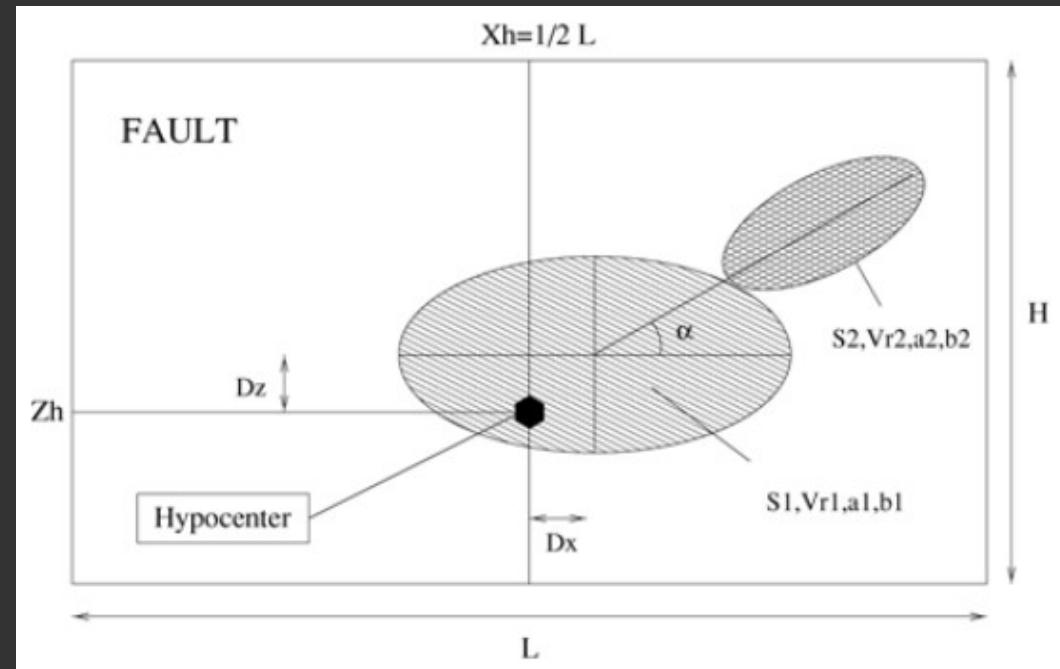
Limitation of iterative decomposition

Emphasize problem
of multi-time-window



P. Ihmlé, 1998

A solution: invert for elliptic slip patches

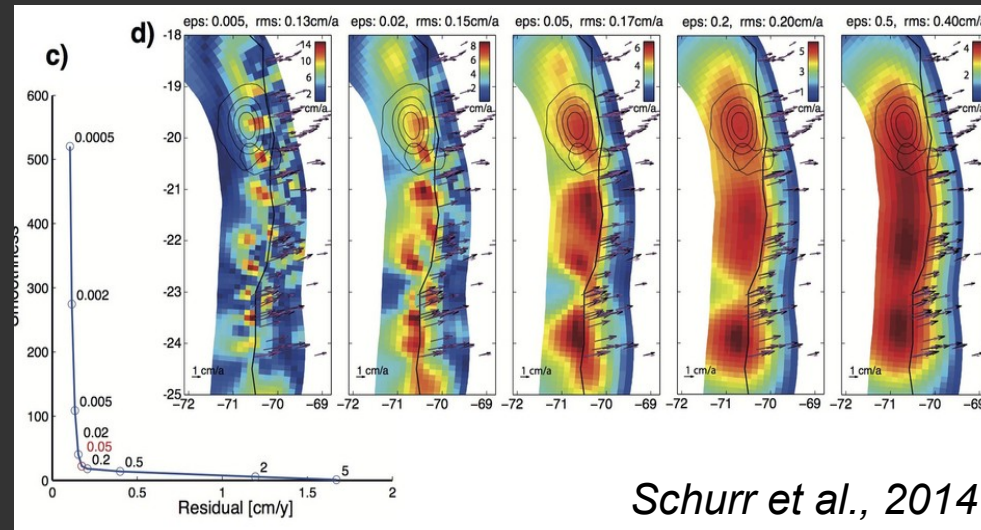


Vallée and Bouchon 2004, Peyrat et al. 2010.
Not adapted for near-field analysis

Regularization

3 main types of regularization: M_0 , spatial and sometimes temporal smoothing. Parametrization is already a way to regularize your problem.

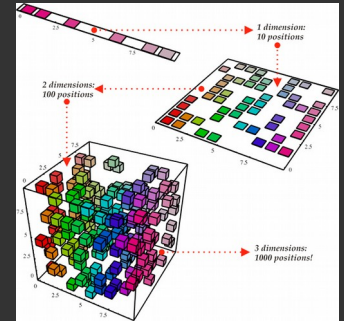
- L-curve: alibi to a subjective choice of spatial smoothing



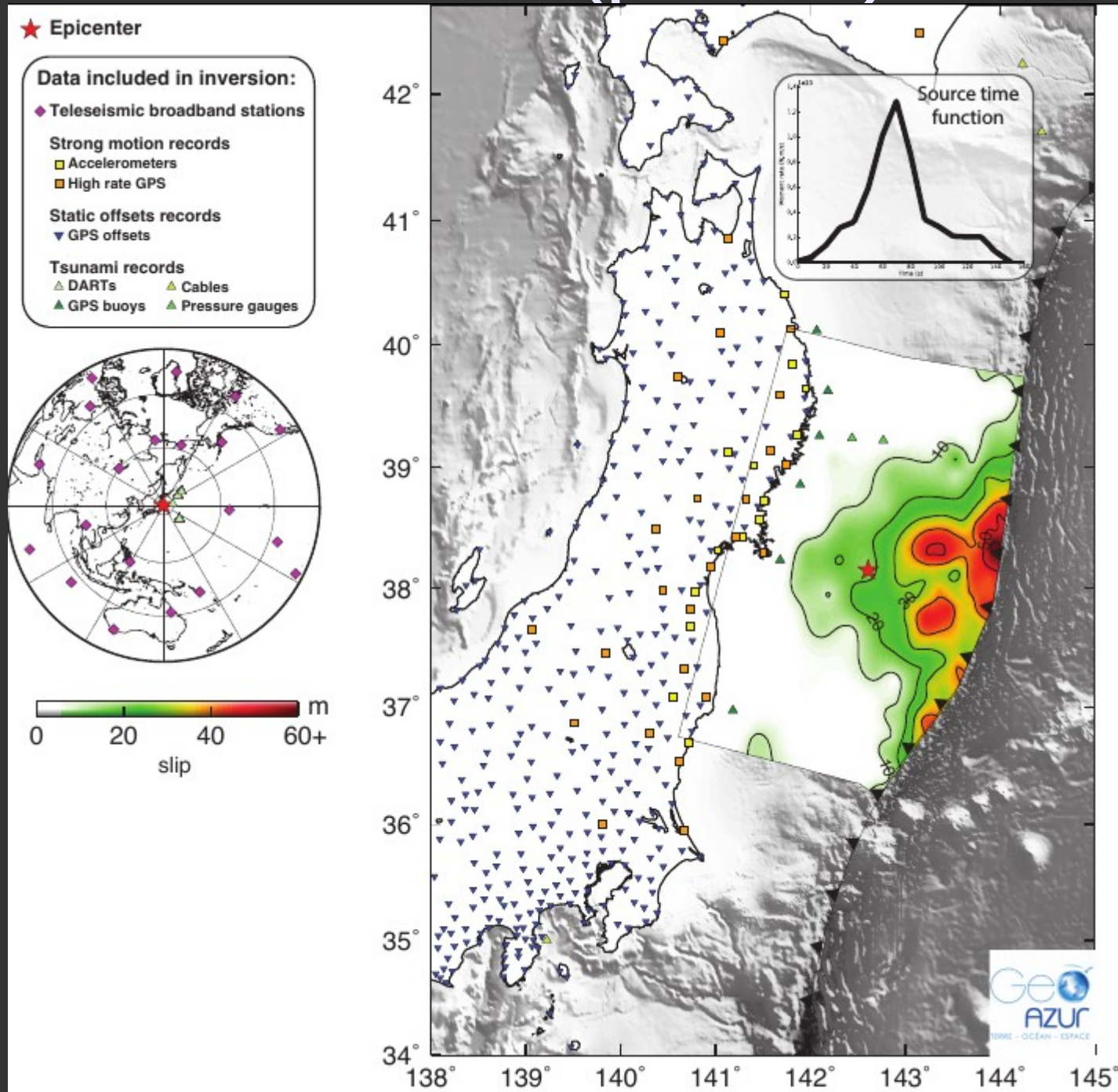
- ABIC : Akaike Bayesian Information Criteria. Regularization with some theoretical basis. Still not perfect, e.g. if non-gaussian uncertainty or positivity constrain (Fukuda&Johnson2008)

How to incorporate other types of data (GPS, tsunami...)?

- linear/linearized: just need to add matrices of each dataset,
- MC inversion: exploration scheme totally independent of type of data. Only limited by number of parameters to invert (the curse of dimensionality)
- How to weight the different datasets ?
 - equal contribution of each dataset: most common strategy,
 - ad-hoc adjustment depending on “quality” of data

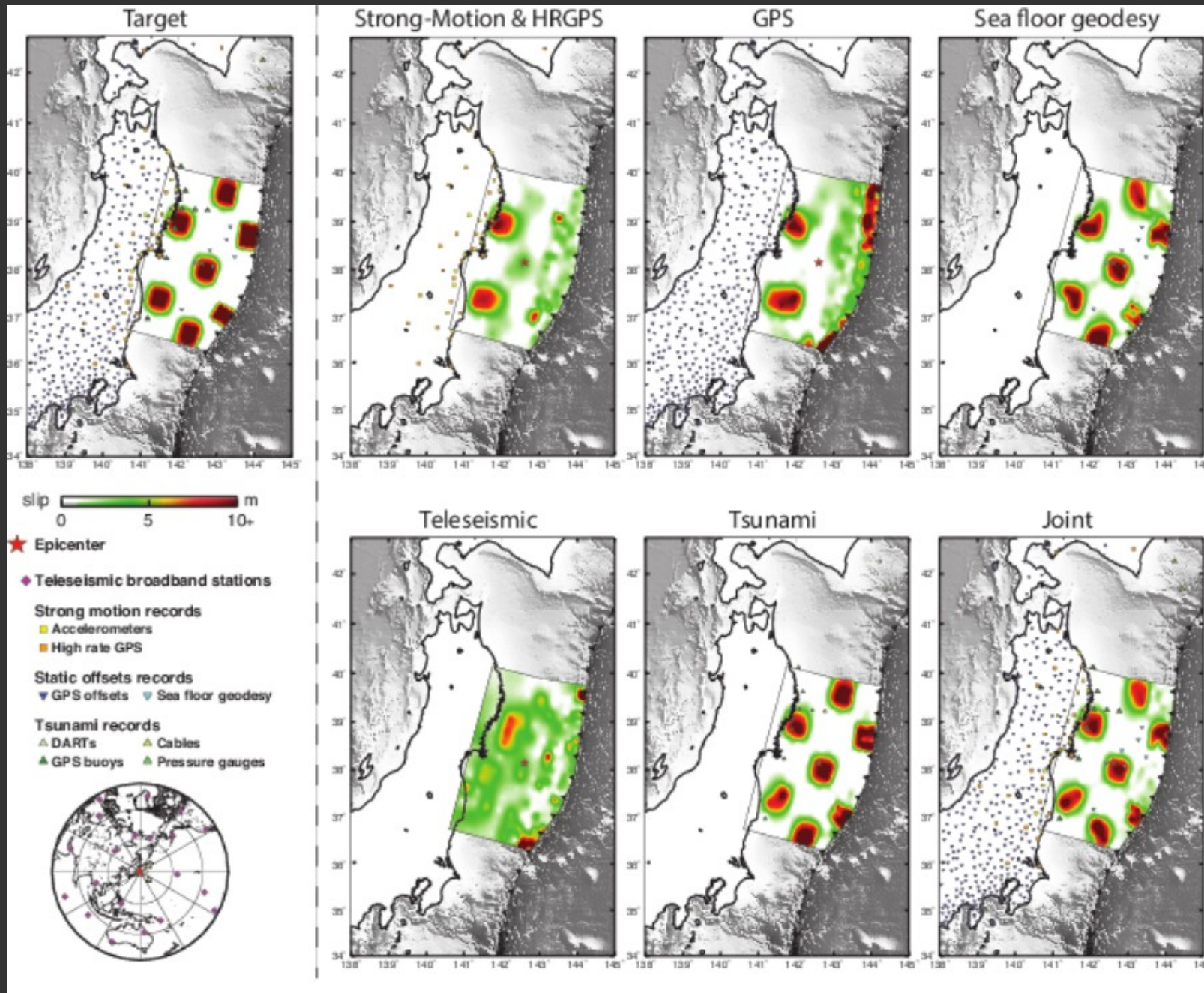


Exemple on Tohoku (poster)

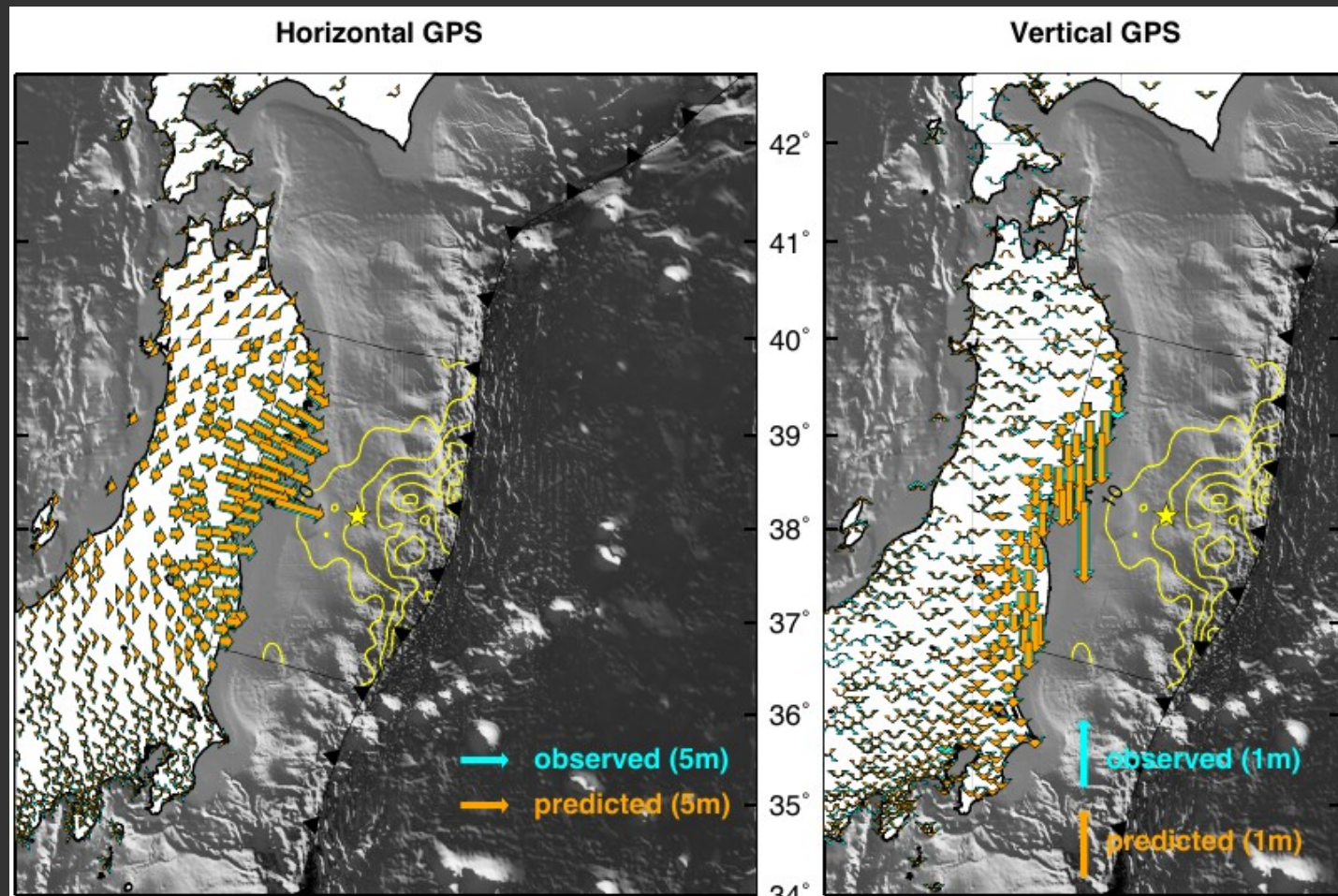


Bletary et al,
2014

Exemple on Tohoku (poster)



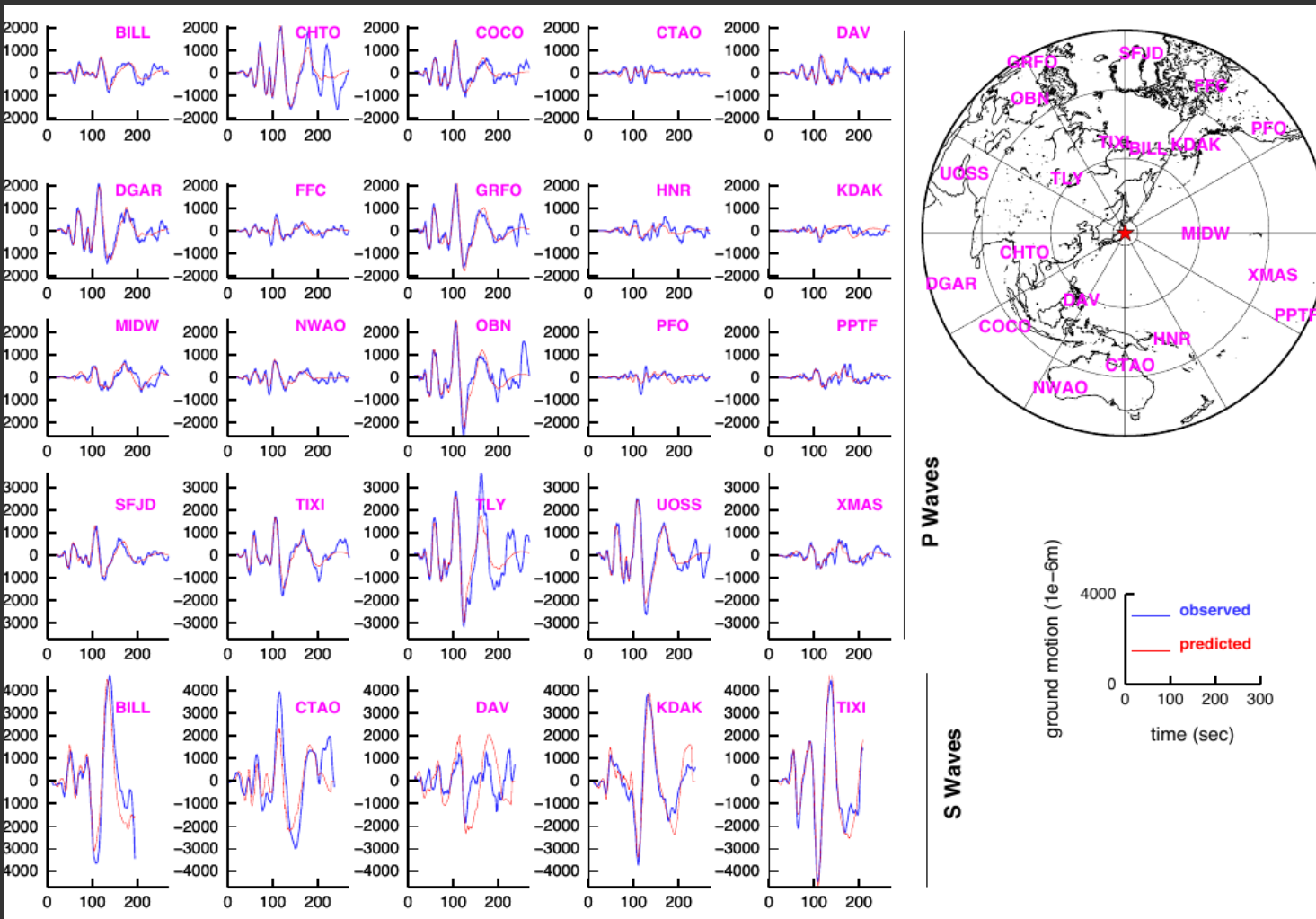
Fit of the data



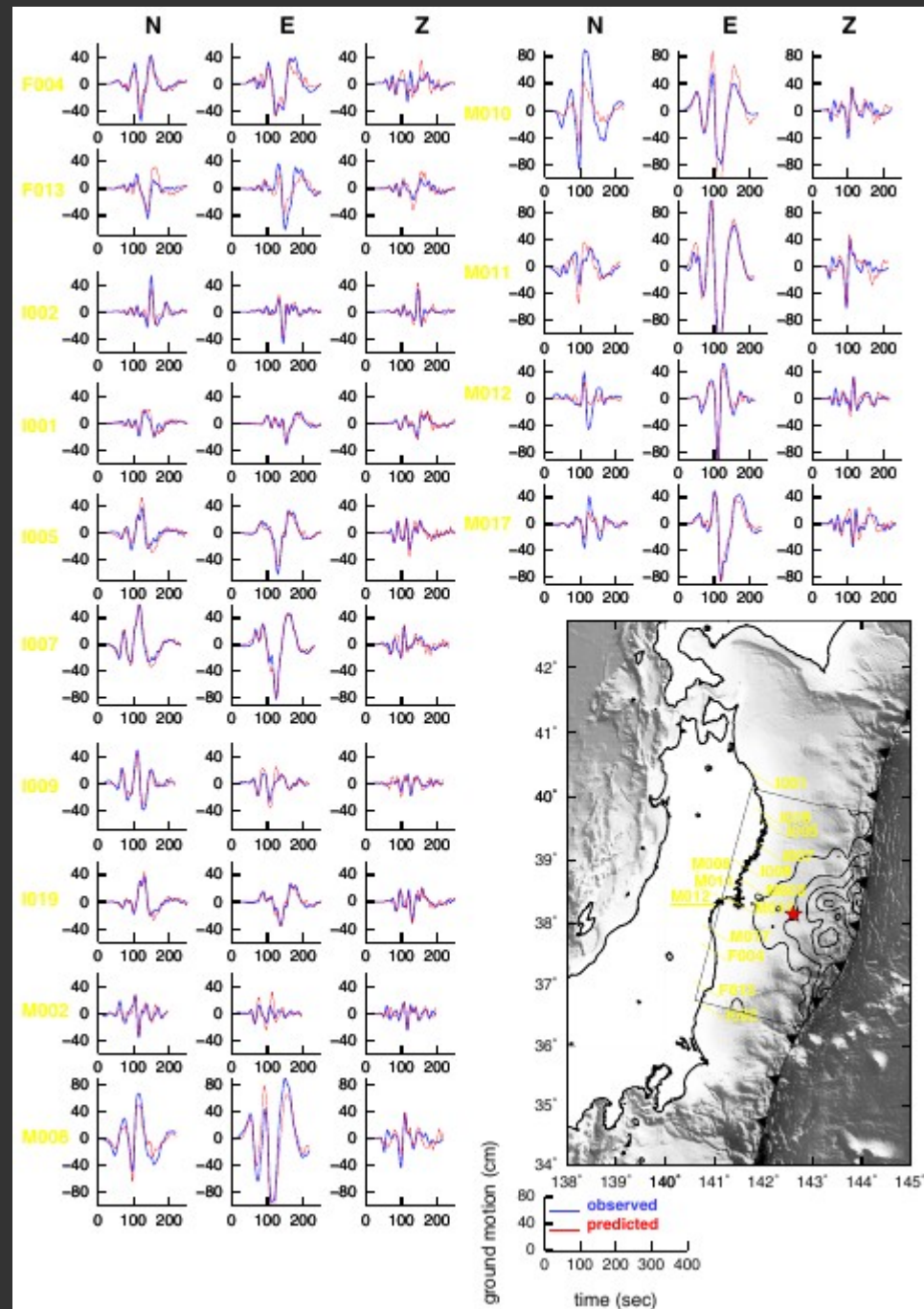
- GPS

Fit of the data

- GPS
- Teleseismic

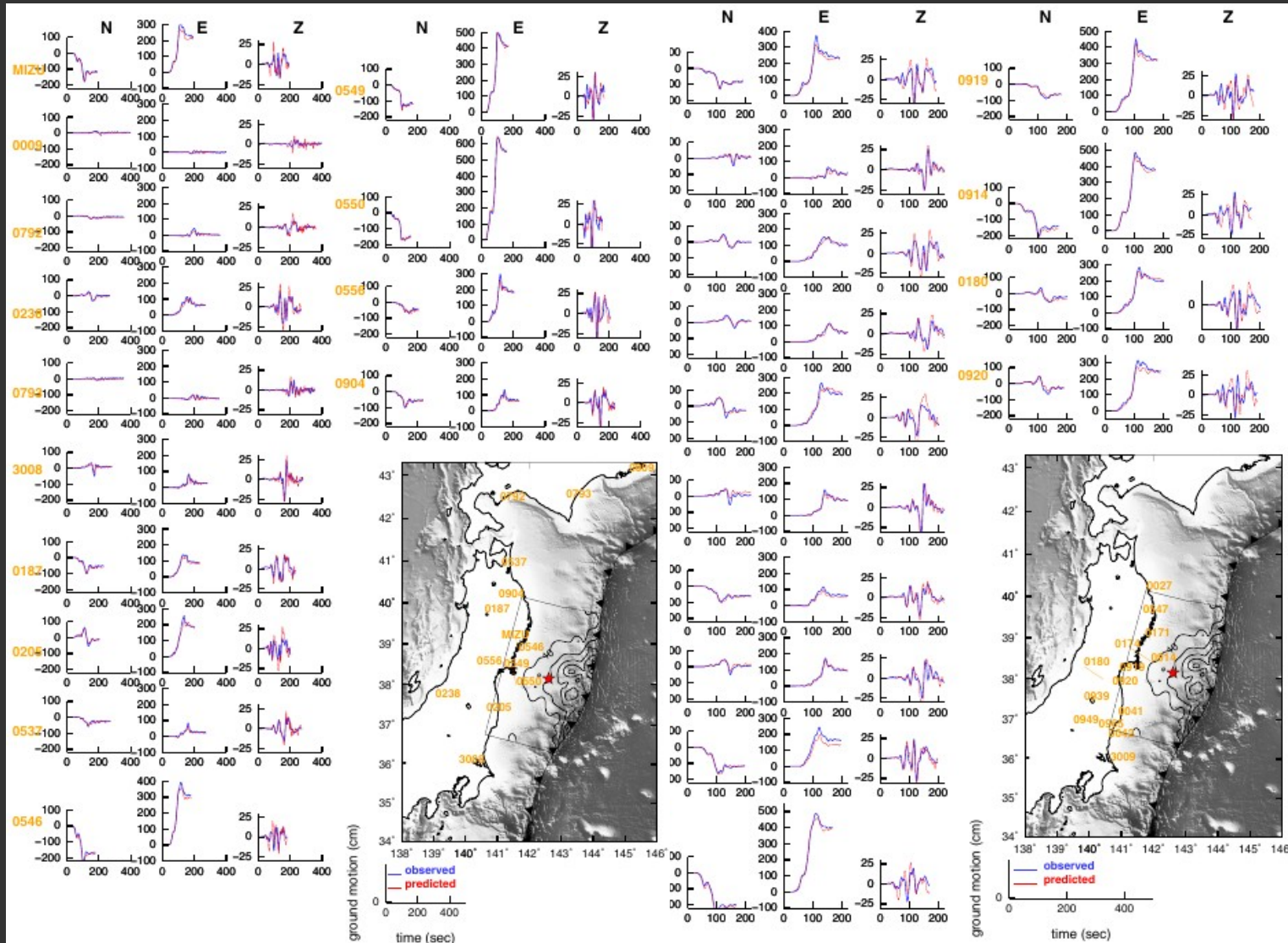


Fit of the data



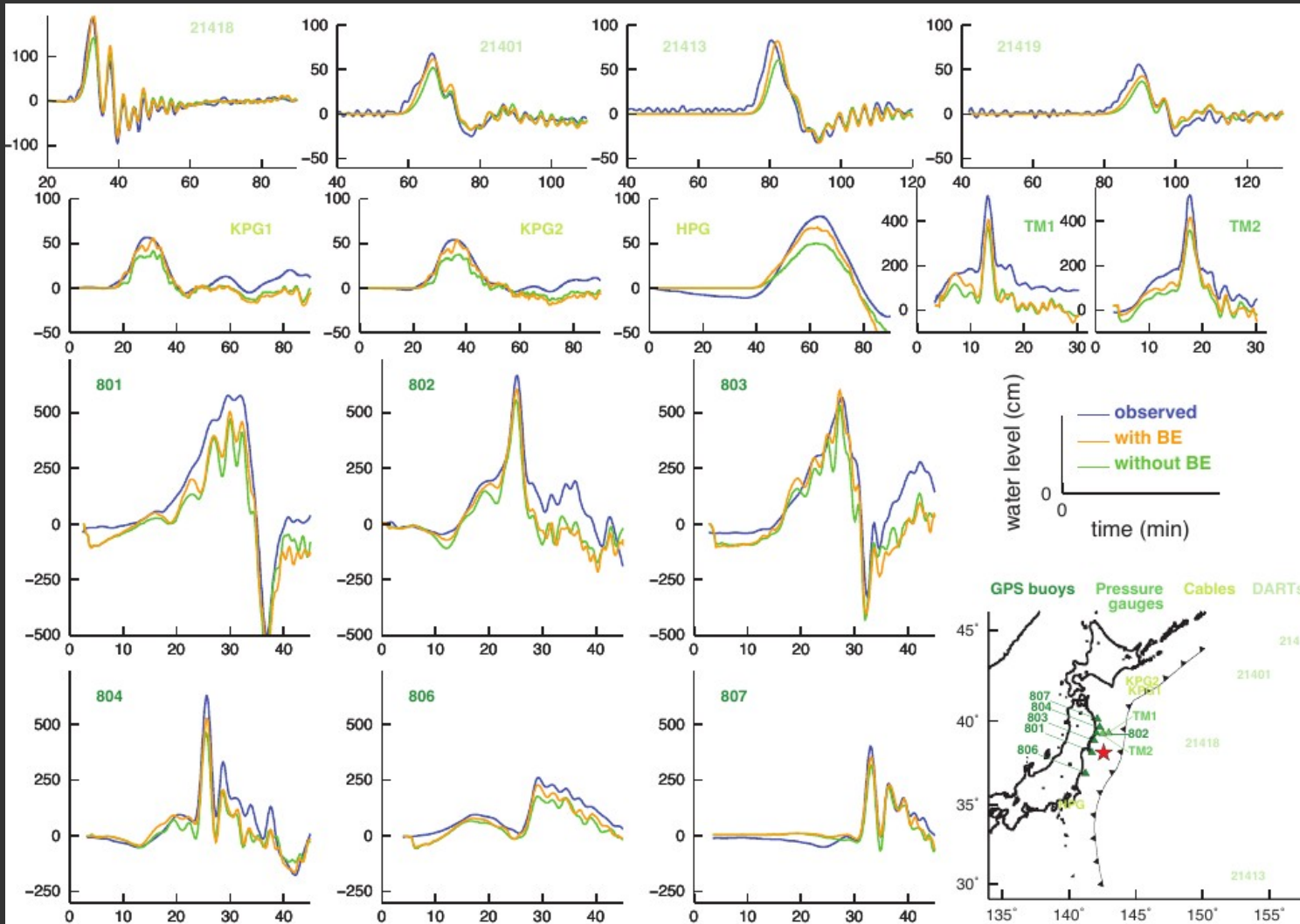
- GPS
- Teleseismic
- Strong-motions

Fit of the data



- GPS
- Teleseismic
- Strong-motions
- Motograms (1sps cGPS)

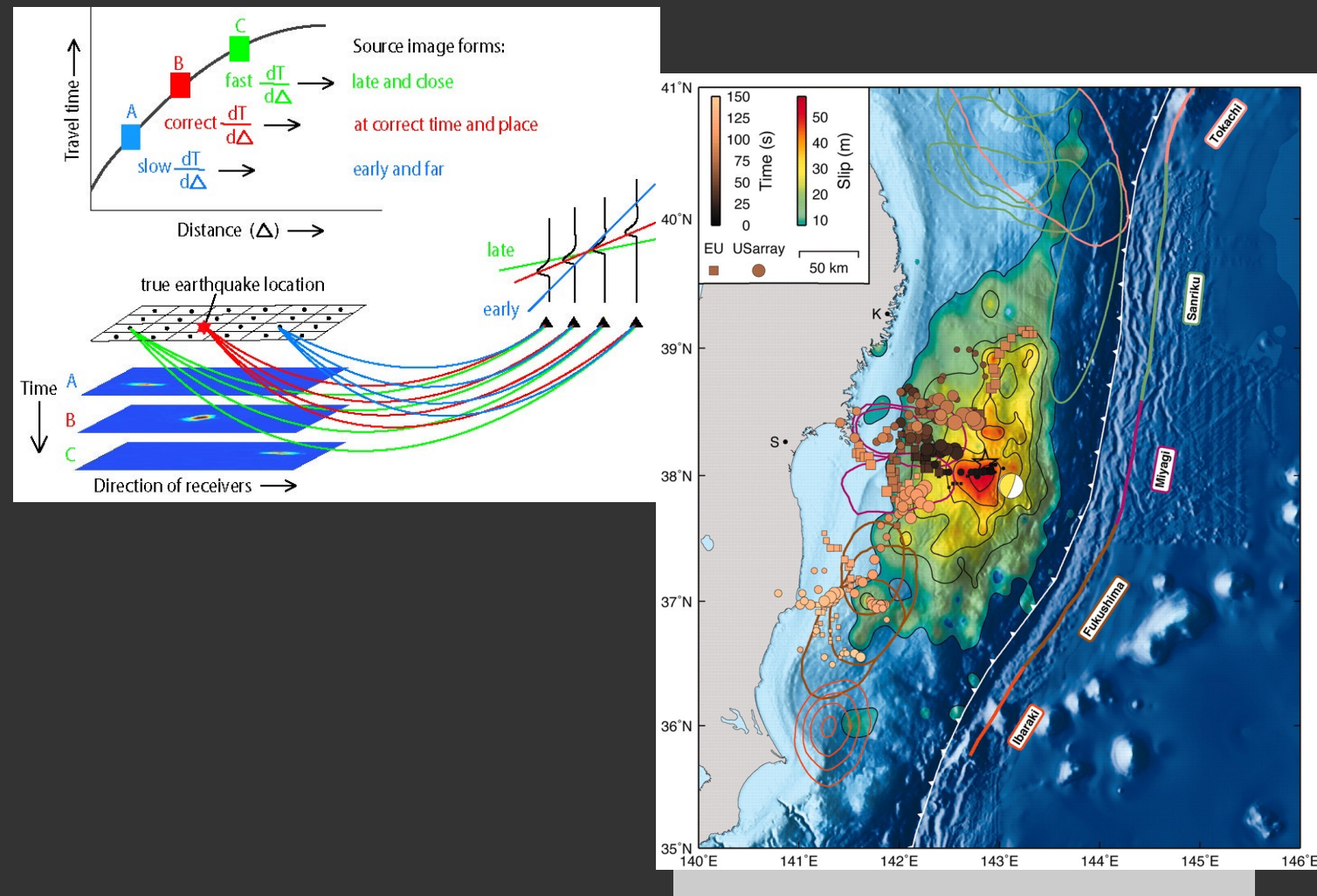
Fit of the data



- GPS
- Teleseismic
- Strong-motions
- Motograms (1sps cGPS)
- Tsunami
only data set a bit difficult to fit

Giant earthquake, crazy datasets... and still, quite easy to fit! 😊

Back-projection



HF radiation \neq high slip patches (Simons et al., 2011)

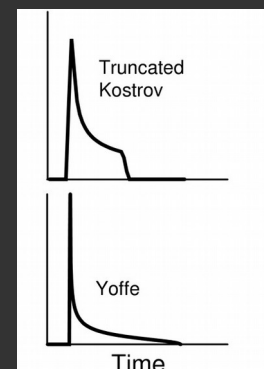
Conclusion

- kinematic inversion means getting space+time information from just time series... it's difficult!
 - seismic data should be inverted jointly with other datasets

...the slip amplitude distribution is similar using either approach, but important differences exist in the rupture propagation models. The single-window method does a better job of recovering the true seismic moment and the average rupture velocity. The multi-window method is preferable when rise time is strongly variable, but tends to overestimate the seismic moment. Both methods work well when the rise time is constant or short compared to the periods modeled. Neither approach can recover the temporal details of rupture propagation unless the distribution of slip amplitude is constrained by independent data.

Cohee and Beroza, 1994

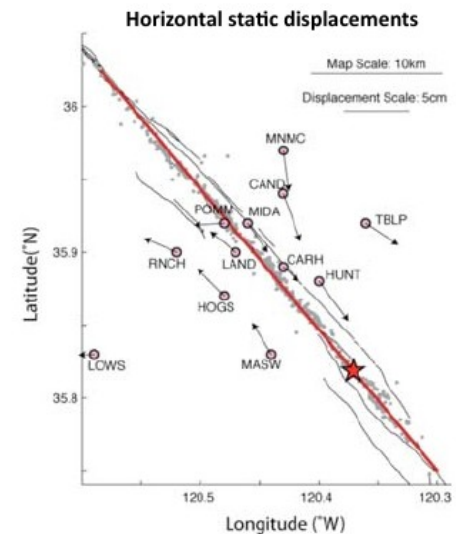
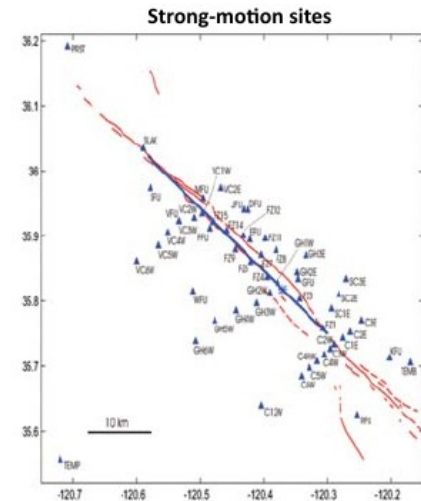
- models based on joint inversion have consistent 1st order features!
- still not able discriminate slip functional, reject repeating slip,
- with good data, fault geometry often becomes a limitation,



Conclusion

- Can we improve resolution?
move to higher freq and 3d... or
do something else,

■ M 6 Parkfield earthquake, very well recorded; velocity structure and fault geometry well known



- Will always be difficult to objectively determine regularization of problem and weighting of the different datasets
 - let's move to Bayesian? no regularization, can put a priori information, get errors/correlations on solution, get families of possible models... and run Tflops simulations

The *Kiwi Tools* package

The Kiwi Tools are an open source software package which allows fast calculation of synthetic seismograms for extended earthquake sources and can be used as a basis for source inversion procedures.

Pyrocko



Pyrocko

Pyrocko is a seismology toolbox and library, written in the Python programming language.

Standalone applications

Besides being usable as a framework for own developments a few standalone tools for everyday seismological practice are contained in Pyrocko:

- **Snuffler**: an [extensible](#) seismogram browser and workbench
- **Cake**: travel-time and ray-path computations for 1D layered earthmodels
- **Fomosto**: a tool to manage pre-calculated Green's function stores
- **Jackseis**: command-line tool for common waveform archive data manipulations

The *rapidinv* program

Rapidinv is a Python script to perform the inversion of point and finite source parameters. Entirely based on the Kiwi Tools, its aim is to simplify the inversion process for the user, still allowing a large flexibility on the inversion setup. Rapidinv requires the prior installation of the Kiwi Tools, the generation of a Green's function database, and the preprocessing of seismic data. Rapidinv performs the source parameter inversion in different steps, first solving the inverse problem for a point source model, and then extending it to the search of finite source parameters. The point source parameter inversion allows for the determination of best double couple and full moment tensor model, and provide information on the apparent duration of the rupture at different stations. The different inversion steps can be either carried out by fitting amplitude spectra or waveforms, and allow a flexible selection of the desired stations, seismogram components, seismic phases, frequency filters, weighting and misfits.



Book Chapter

A practical on moment tensor inversion using the Kiwi tools



Released

Cesca, S., Heimann, S. (2013): A practical on moment tensor inversion using the Kiwi tools. - In: Bormann, P. (Ed.), *New Manual of Seismological Observatory Practice 2 (NMSOP-2)*, Potsdam : Deutsches GeoForschungsZentrum GFZ, p. 1-24.

DOI: https://doi.org/10.2312/GFZ-NMSOP-2_Fk_3.6

Some references

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- M. Mai Earthquake rupture inversions : a primer, QUEST meeting 2011
- G. Festa and A. Zollo From data to source parameters: Kinematic modeling, in The Mechanics of Faulting: From Laboratory to Real Earthquakes, 2012

Books

- Udias, Madariaga and Buforn Source mechanisms of earthquakes, Cambridge Univ. Press, 2014
- Lay and Wallace, Modern Global Seismology, Academic Press, 521pp, 1995

You're welcome to endure the hard working conditions in South of France...

- Want to come for a seminar?
- Want to come for internship, PhD, postdoc?

