Earthquake Geodesy
Modelling Surface Displacements measured by GPS and InSAR

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Agenda

1. Part A: The Forward Model
   Representation Theorem
   Rectangular dislocation
   Exercises

2. Part B: Optimization of observed surface displacements
   This non-linear problem
   The model parameter space
   Exercises

3. Additional Information
   Data Sources and Processing Software for GPS and InSAR

4. Literature
The birth of modern seismology

First observations of earthquake surface faulting after the 1906 Great San Francisco earthquake

Surface rupture of the 1906 San Francisco earthquake

Horizontal offset

USGS historical picture database
Fault slip types

Left-lateral strike-slip fault
($\lambda = 0^\circ$)

Right-lateral strike-slip fault
($\lambda = 180^\circ$)

Normal dip-slip fault
($\lambda = -90^\circ$)

Reverse dip-slip fault
($\lambda = 90^\circ$)

Stein & Wyssesion, 2003
The seismic cycle from a bird’s view

From fast to slow motion

(a) Full relaxed status at $t = 0$

(b) Continental drift, fault loading, interseismic state

(c) Co-seismic rupture, fault unloading
Representation Theorem for earthquake faulting

Volterra’s Theorem

\[
\begin{align*}
  u_n(\vec{x}, t) &= \int_{-\infty}^{\infty} d\tau \int_{\Sigma} \int \left[s_i(\xi, \tau)\right] \cdot c_{ijpq} \cdot \nu_j \cdot \frac{\partial}{\partial \xi^q} G_{np}(\vec{x}, t-\tau, \xi, 0) d\Sigma
\end{align*}
\]

- Forward modeling: With a given rupture process we can predict displacements at any point in/on the Earth
- Inversion: With a given surface displacement, we can infer the rupture process
- Geodetic data: Neglect the temporal resolution and look at the total displacement only: \( \int_{t_1}^{t_2} u_A dt \)
Representation Theorem for earthquake faulting

Volterra’s Theorem

\[
\mathbf{u}_n(\mathbf{x}, t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} \left[ s_i(\xi, \tau) \right] \cdot c_{ijpq} \cdot \mathbf{v}_j \cdot \frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t-\tau, \xi, 0) d\Sigma
\]

- Displacement at location \( \mathbf{x} \)
- Time-dependent slip on fault plane
- Elasticity tensor normal to the fault plane
- Green's function: system response in \( n \)-direction due to unit impulse in \( p \)-direction on the fault plane

- Forward modeling: With a given rupture process we can predict displacements at any point in/on the Earth
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Representation Theorem for earthquake faulting

Volterra's Theorem

\[ u_n(\vec{x}, t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} \left[ s_i(\xi, \tau) \right] \cdot c_{ijpq} \cdot \nu_j \cdot \frac{\partial}{\partial \xi_q} G_{np}(\vec{x}, t - \tau, \xi, 0) d\Sigma \]

- **Forward modeling**: With a given rupture process we can predict displacements at any point in/on the Earth
- **Inversion**: With a given surface displacement, we can infer the rupture process
- **Geodetic data**: Neglect the temporal resolution and look at the total displacement only: \( \left( \int_{t_1}^{t_2} u_A dt \right) \)
Near-field and far-field of a rupture/dislocation source

- Near-field sampling by using InSAR, displacements $\sim M_0$
- Far-field sampling by using teleseismic data, displacements $\sim M_0$

Depth: ~20 km
Horizontal extent: 10-100 km
Rupture model parameters
Rupture model parameters

- **Dimension**
  1. length [km]
  2. width [km]
  3. depth [km]

- **Orientation**
  4. dip from hor. [°]
  5. strike from North [°]

- **Location**
  6. x/East [km]
  7. y/North [km]

- **Slip**
  8. strike slip [m]
  9. dip slip [m]
  10. opening [m]
Rupture model parameters

Dimension

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Slip

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Dislocation model
How to model dislocations in Matlab

\[ U = \text{disloc}(mp, xloc, nu) \]

- \( U \) \( 3 \times n \) displacement vector (ENU)
- \( mp \) model parameter: \( 10 \times 1 \) vector with source dimension (length, width, depth)
- source orientation (dip, strike)
- source location (\( x, y \)) and slip definition (s-slip, d-slip, opening)
- \( xloc \) \( 3 \times n \) vector with \( n \) observation points (ENU)
- \( nu \) Poisson’s ratio (elasticity)

- Code written by Peter Cervelli, based on Okada (1985), purely elastic
- slip input = deformation output (usually \([m]\)), source geometry: \([km]\)
- Start with a Poisson’s ratio of 0.25
Deformation caused by a vertical strike-slip fault

Surface rupture (or co-seismic)

Buried fault (or inter-seismic)
Exercises - Displacing the surface

or “producing interferograms”

**Purpose:** Get a feeling for slip on faults and the induced surface displacements.

On your virtual machines:
Go to /home/jissy/Documents/2_Tuesday/EQ_geodesy/fwdtools_octave
Open Dislocation_fwd.m

**Try out “randomly”:**
edit lines “mp = [...]” and set up different source mechanisms

For the special cases:
**Iceland:** edit LOS to \( \text{LOS} = [-0.4 \ 0.1 \ -0.9] \)

**Nepal:** edit \( \lambda = 0.23/2 \) and edit \( \text{LOS} = [0.5 \ 0.1 \ -0.7] \)
Exercise - Special Task A

**Voluntarily - specific tasks:** Reproduce interferograms

A: the 2000 Kleifarvatn earthquake, Iceland, Mw5.9

(we will try to optimize this earthquake fault after lunch...)

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![Interferogram Images](image-url)
Exercise - Task B

B:
The 2015 Nepal earthquake, Mw7.8
Source scaling for earthquakes

“physics” - not all model parameter combination possible are realistic or observed in nature

Magnitude and Seismic Moment

Moment Magnitude (Aki & Richards) \[ M_w = \frac{2}{3} \log(M_0) - 6.03 \]

Seismic Moment [Nm] \[ M_0 = \text{rigidity}(30\text{GPa}) \cdot \text{slip} \cdot \text{length} \cdot \text{width} \]
Deviations in scaling relations for different faulting styles:

For strike-slip earthquakes and increasing moment:
- the fault length grows faster than fault width (fault width is even saturated)
- the fault area grows slower than for earthquakes on average
- the slip grows faster than for earthquakes on average

For dip-slip earthquakes and increasing moment the fault length and width grow similarly.
Disloc parameter conventions as right-hand law

- Right hand is foot wall, the palm fault plane (dip values 0 to -90 deg, negative only)
- Thumb is fault's upper edge
- Thumb is pointing in strike direction
- Fingers point in dip-slip direction
- Thumb points in dextral strike slip

Location (x,y,z)

- Positive dip-slip: normal faulting
- Negative dip-slip: thrust faulting
- Positive strike-slip: right-lateral slip
- Negative strike-slip: left-lateral slip

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Case at hand: The 2000 Kleifarvatn earthquake

Sudhaus & Jonsson, 2009
The non-linearity in the dislocation modelling

Linear dependency:
Surface displacements for a vertical plane with varying slip

Non-linear dependency:
Surface displacements for a vertical plane with varying strike.
The non-linear inverse problem solved with “Direct Search”

The objective function

misfit (or ”cost“) function as a special objective function

L₂-norm: misfit \( \Phi(m) = \|d_{obs} - d_{synth}\|_2 = \sqrt{(d_{obs} - d_{synth})^2} \)

... very sensitive. Best suiting for highly non-linear problems.

The misfit function \( \Phi(m) \) is a function of \( m \) since \( d_{synth} = G(m) \).

\[
\Phi(m) = \|d_{obs} - d_{synth}\|_2 = \sqrt{(d_{obs} - d_{synth})^2} = \sqrt{r^T r}, \text{ with } r \text{ the residual.}
\]

Considered the data are noisy and/or are correlated we can apply a weighting
in the misfit function to account for that:

\[
\Phi(m) = \sqrt{(Wr)^T (Wr)}, \text{ with } W \text{ being a of weighting vector, or}
\]

\[
\Phi(m) = \sqrt{r^T \Sigma^{-1} r}, \text{ with } \Sigma \text{ being the data error covariance matrix.}
\]
The model parameter space

- is spanned by all (N) model parameters and is therefore N-dimensional.
- it is finite, however, given physical constraints and prior information.
Exercises - Direct Search #2 (automatically)

**Purpose:** Implementations and trying-out of simple Monte Carlo Optimizations.

On your virtual machines:
Go to `/home/jissy/Documents/2_Tuesday/EQ_geodesy/optitools_octave`
Open `Nonlin_Kleifar_course_1.m`

Editing:
**Setting model parameter bounds:** edit lines 99 & 100
**Setting optimization options for evolution:** edit line 148

`run Nonlin_Kleifar_course_1` in octave

`run Nonlin_Kleifar_course_2` in octave

or (for Simulated Annealing)

**Setting optimization options:** uncomment line 156
in `Nonlin_Kleifar_course_2.m`: comment line 9, uncomment line 11

`run Plot_Model_Parameters.m` and `Plot_Model_Predictions.m` to see your results.
Exercises - Two Monte Carlo Flow Charts

Flow of an Evolutionary Algorithm

A) **Recombine** parent model parameters (○) to new models (o, offspring).

B) **Mutate** new models by adding normal-distributed small numbers.

C) **Select** best-fit new models and form a new parent generation. (& back to A)

Flow of Simulated Annealing

A) select new models \( m_x \) near \( m_n \) by only varying \( a \) in \( m_n \)

B) evaluate all \( m(a_x,b_n) \) and pick from these randomly the new \( m \) (Eq. 3):
small-misfit models are more likely to be picked than high-misfit models.

C) select new models \( m_x \) near \( m_{n+1} \) by only varying \( b \) in \( m_{n+1} \)

D) evaluate all \( m_x \) and pick new \( m \). Before starting at A) again maybe lower temperature.
Extra Information on the following slides ...
Satellite missions: Time Lines and Specifications
SAR Roundtable, January 8, 2014

Mission Specifications

<table>
<thead>
<tr>
<th>Name/Operated by</th>
<th>ERS-1/2, Envisat, Sentinel</th>
<th>TerraSAR-X, TanDEM-X</th>
<th>JERS &amp; ALOS-1/2</th>
<th>Radarsat-1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revisit times</td>
<td>ERS-1/2, Envisat: 36 days (future) Sentinel-1: 12 days</td>
<td>TerraSAR-X &amp; TanDEM-X: 11 days (future) TanDEM-L: no info found</td>
<td>JERS &amp; ALOS-1: 36 days (future) ALOS-2: 14 days</td>
<td>Radarsat-1/2: 24 days (future) Radarsat Constellation: 4 days</td>
</tr>
<tr>
<td>Band</td>
<td>C-band</td>
<td>X-band</td>
<td>L-band</td>
<td>C-band</td>
</tr>
<tr>
<td>Coverage</td>
<td>Huge data set with worldwide coverage</td>
<td>Coverage only on demand - no background mission!</td>
<td>Global coverage with a concentration in Asia</td>
<td>Focus on permafrost/ice covered globe, good coverage over northern hemisphere and science hot spots, e.g. Iceland, Hawaii</td>
</tr>
<tr>
<td>Data availability (for research)</td>
<td>More or less freely available for registered users via EOLI-SA</td>
<td>Research scene charges are about 400 EUR, for researchers with a proposal at DLR a limited amount of data is free - check for opportunities (in particular for GFZ access is easier, in case ask T. Walter or M. Matzr)</td>
<td>JERS: normal charges ~19 US $ per scene, ALOS: Access via EoUSA for some (l) areas Or (j) ERS &amp; ALOS) for free for researchers with an accepted proposal (check for opportunities* at <a href="http://www.eorc.eo-usa.mtv.de/opus/scheduled.htm">http://www.eorc.eo-usa.mtv.de/opus/scheduled.htm</a></td>
<td>Available for paying customers at a charge of 3000 - 4000 EUR per scene, or some few scenes (~200) via research proposals (check for opportunities* at <a href="http://www.asc-csa.gc.ca/eng/aodefault.asp">http://www.asc-csa.gc.ca/eng/aodefault.asp</a>)</td>
</tr>
<tr>
<td>Vena</td>
<td>ERS-1, 2: 1990 - 2000; ERS-2: 2000 - 2010; Sentinel-1: 2014 -</td>
<td>More than 100 km (200 km for Envisat wideswash data)</td>
<td>Ground resolution: 4 m in azimuth, 2 m in range. Swath with ~70 km</td>
<td></td>
</tr>
</tbody>
</table>

Different penetration depths:

- B-SAR: 3 cm
- C-SAR: 5.6 cm
- X-SAR: 3.1 cm
- L-SAR: 24 cm

Satellite missions:

EROS-1/2, Envisat, Sentinel
- C-band: 5.6 cm
- L-band: 24 cm

TerraSAR-X, TanDEM-X
- X-band: 3.1 cm
- L-band: 24 cm

JERS & ALOS-1/2
- X-band: 3.1 cm
- C-band: 5.6 cm

Radarsat-1/2
- C-band: 24 cm

Different penetration depths:

- B-SAR: 3 cm
- C-SAR: 5.6 cm
- X-SAR: 3.1 cm
- L-SAR: 24 cm

Data Sources and Processing Software for GPS and InSAR

H. Sudhaus
Earthquake Geodesy
Apr 21, 2015 24 / 29
GPS Data

Data Sources:

- from collaborators or your own network
- from published work
- https://unavco.org/data/data.html

Processing software:

- e.g. **GAMIT**, consult unavco page above
  - **Bernese**: www.bernese.unibe.ch/
- https://unavco.org/data/data.html
InSAR Processing software

- **GMTSAR**: InSAR processing system based on GMT (Scripps/San Diego, Hawaii)
- **DORIS**: Delft object-oriented radar interferometric software (TU Delft)
- **ROI_PAC**: Similar to GAMMA, deprecated
- **ISCE**: InSAR Scientific Computing Environment (JPL, Caltech, Stanford), ROI_PAC offspring
  - **GAMMA**: commercial, GAMMA Remote Sensing AG, Switzerland
  - **SARscape**: commercial, but the only software with a graphical user interface, based on ENVI, SARmap SA, Switzerland

...
InSAR Time-series solver

- **StaMPS**: Stanford Method of Persistent Scatterers, developed by Andy Hooper (Stanford U, TU Delft, Leeds U), based on DORIS output, PS technique
- **π-RATE**: (Poly-Interferogram Rate And Time-series Estimator), Matlab toolbox developed by Biggs, Elliott, Wang (Leeds U), uses full interferograms processed with ROI_PAC
- **GIAnT**: Generic InSAR Analysis Toolbox, based on ISCE
  - **IPTA** (GAMMA): Interferogram Point Target Analysis, PS technique
  ...
Literature on deformation modeling

Chapters:

- Deformation, Stress, Conservation Laws
- Dislocation Models of Strike-slip Faults
- Dip-Slip Faults and 3D Dislocations
- Crack Models of Faults
- Elastic Heterogeneity
- Postseismic Relaxation
- Volcano Deformation
- Topography and Earth Curvature
- Gravitational Effects
- Poroelastic Effects
- Fault Friction
- Interseismic Deformation

(library at Princeton Press, ~90 EUR)
Literature on deformation modeling

On moodle: