Earthquake Geodesy

Modelling Surface Displacements measured by GPS and InSAR

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Agenda

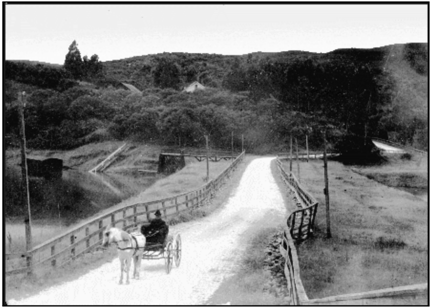
1. Part A: The Forward Model
   - Representation Theorem
   - Rectangular dislocation
   - Exercises
2. Part B: Optimization of observed surface displacements
   - This non-linear problem
   - The model parameter space
   - Exercises
3. Additional Information
   - Data Sources and Processing Software for GPS and InSAR
4. Literature

The birth of modern seismology

First observations of earthquake surface faulting after the 1906 Great San Francisco earthquake

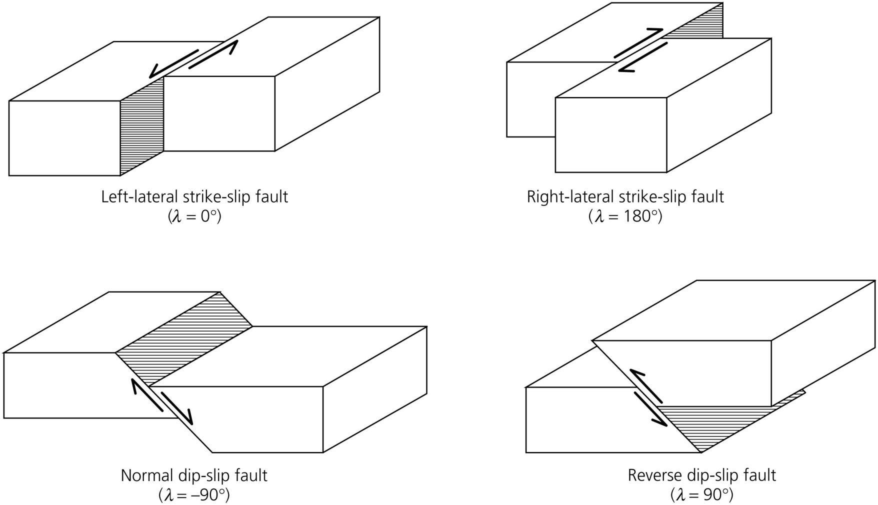
Surface rupture of the
1906 San Francisco earthquake

horizontal offset



USGS historical picture database

Fault slip types

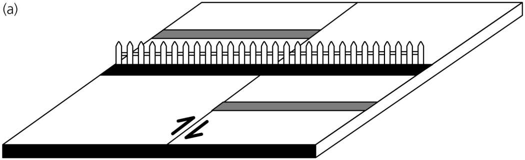


Stein & Wyssesion, 2003

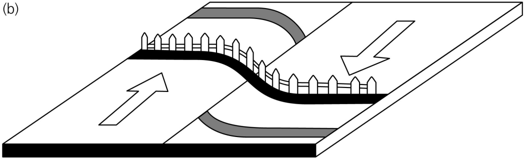
The seismic cycle from a bird's view

From fast to slow motion

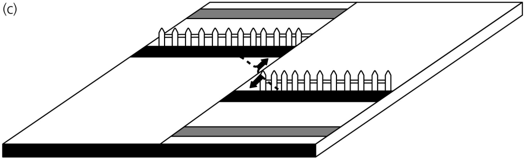
(a)



(b)



(c)



- a Full relaxed status at $t = 0$
- b Continental drift, fault loading, interseismic state
- c co-seismic rupture, fault unloading

Representation Theorem for earthquake faulting

Volterra's Theorem

$$u_n(\vec{x},t)=\int_{-\infty}^{\infty} d\tau \int_{\Sigma}\int [s_i(\xi,\tau)]\cdot c_{ijpq}\cdot \nu_j\cdot \frac{\partial}{\partial \xi_q} G_{np}(\vec{x},t-\tau,\xi,0)\, d\Sigma$$

- Forward modeling: With a given rupture process we can predict displacements at any point in/on the Earth
- Inversion: With a given surface displacement, we can infer the rupture process
- Geodetic data: Neglect the temporal resolution and look at the **total** displacement only: $(\int_{t_1}^{t_2} \mathbf{u_A}\, dt)$

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Displacement at location x

Time-dependent slip on fault plane

Elasticity tensor normal to the fault plane

Green's function: system response in n-direction due to unit impulse in p-direction on the fault plane

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Representation Theorem for earthquake faulting

Volterra's Theorem

Integration over fault plane

Integration over rupture time

Integration over moving particle

$$u_n(\vec{x},t)=\int_{-\infty}^{\infty} d\tau \int_{\Sigma} \int [s_i(\xi,\tau)]\cdot c_{ijpq}\cdot \nu_j \cdot \frac{\partial}{\partial \xi_q} G_{np}(\vec{x},t-\tau,\xi,0)\, d\Sigma$$

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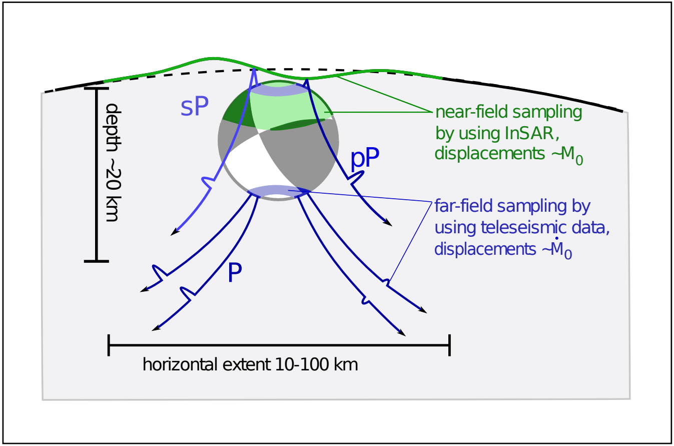
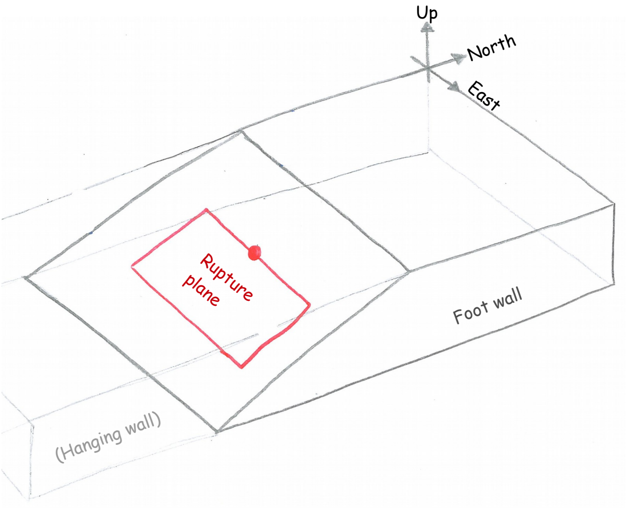
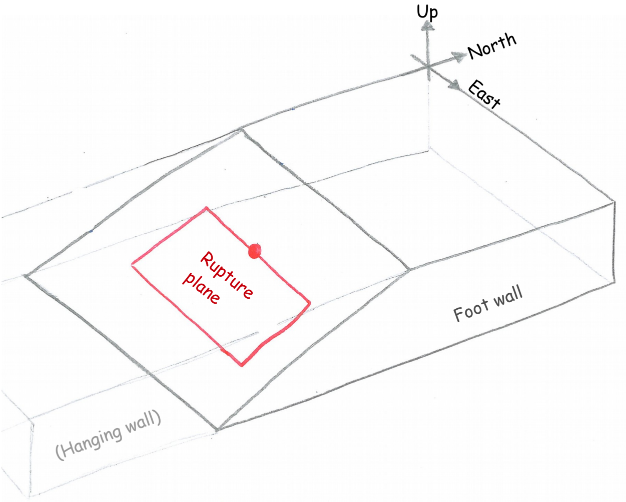
Near-field and far-field of a rupture/dislocation source



Rupture model parameters



Rupture model parameters



Dimension

1. length [km]
2. width [km]
3. depth [km]

Orientation

4. dip from hor. [°]
5. strike from North [°]

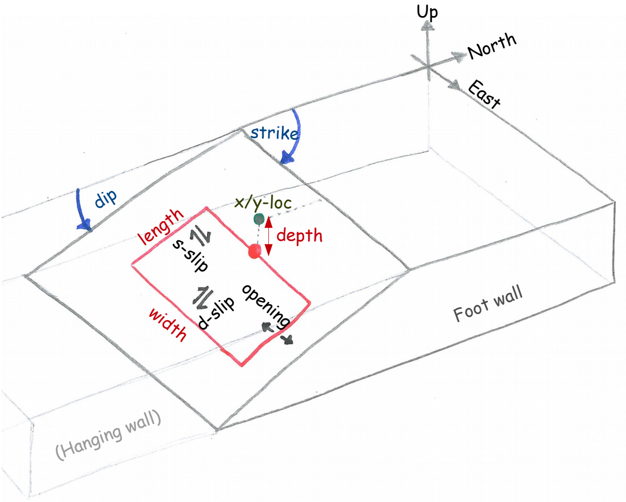
Location

6. x/East [km]
7. y/North [km]

Slip

8. strike slip [m]
9. dip slip [m]
10. opening [m]

Rupture model parameters



Dimension

1. length [km]
2. width [km]
3. depth [km]

Orientation

4. dip from hor. [°]
5. strike from North [°]

Location

6. x/East [km]
7. y/North [km]

Slip

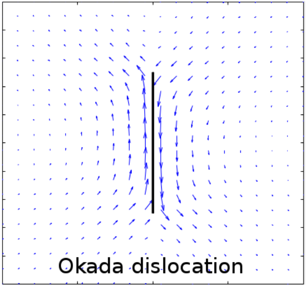
8. strike slip [m]
9. dip slip [m]
10. opening [m]

Dislocation model

How to model dislocations in Matlab

U = disloc(mp,xloc,nu)

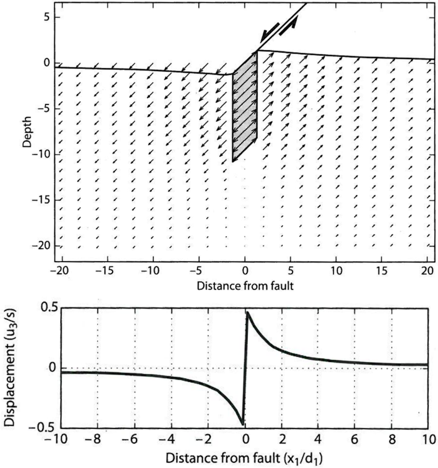
- U: 3×n displacement vector (ENU)
- mp: model parameter: 10×1 vector with
  - source dimension (*length*, *width*, *depth*)
  - source orientation (*dip*, *strike*)
  - source location (x, y) and
  - slip definition (*s-slip*, *d-slip*, *opening*)
- xloc: 3×n vector with n observation points (ENU)
- nu: Poisson's ratio (elasticity)



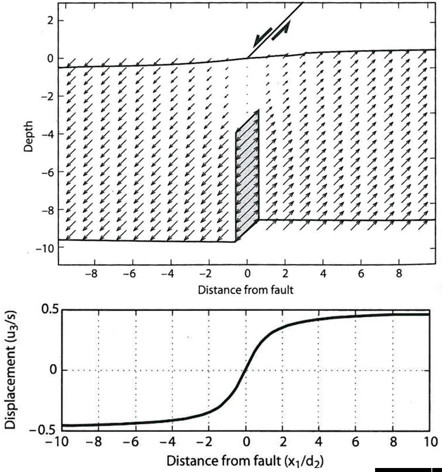
- Code written by Peter Cervelli, based on Okada (1985), purely elastic
- slip input = deformation output (usually [m]), source geometry: [km]
- Start with a Poisson's ratio of 0.25

Deformation caused by a vertical strike-slip fault

Surface rupture (or co-seismic)



Buried fault (or inter-seismic)



Segall, 2010

Exercises - Displacing the surface

or “producing interferograms”

Purpose: Get a feeling for slip on faults and the induced surface displacements.

On your virtual machines:
Go to **/home/jissy/Documents/2_Tuesday/EQ_geodesy/fwdtools_octave**
Open **Dislocation_fwd.m**

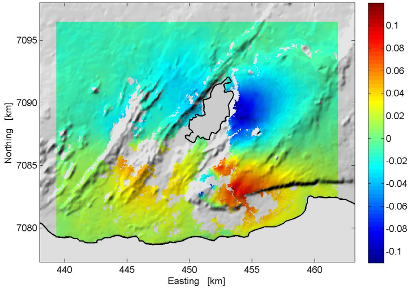
Try out “randomly”:
edit lines “mp = [...]” and set up different source mechanisms

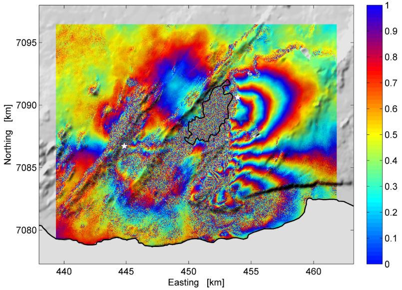
For the special cases:
Iceland: edit LOS to LOS = [-0.4 0.1 -0.9]

Nepal: edit lambda= 0.23/2 and edit LOS = [0.5 0.1 -0.7]

Exercise - Special Task A

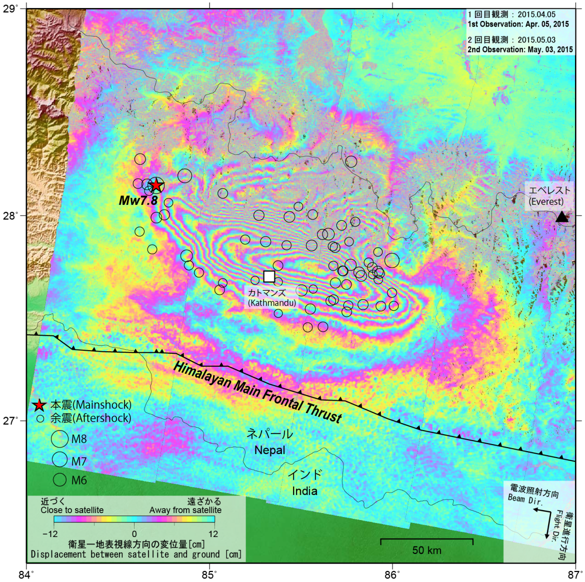
Voluntarily - specific tasks: Reproduce interferograms
A: the 2000 Kleifarvatn earthquake, Iceland, Mw5.9
(we will try to optimize this earthquake fault after lunch...)





Exercise - Task B

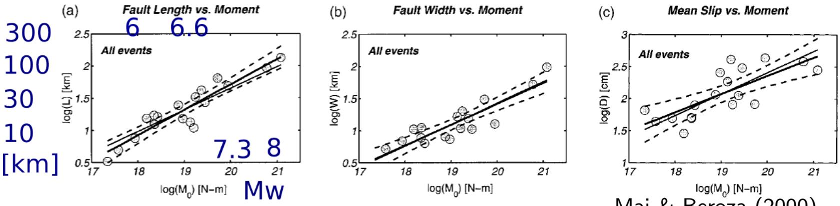
B:
The 2015 Nepal earthquake, Mw7.8



Analysis by GSI from ALOS-2 raw data of JAXA

Source scaling for earthquakes

"physics" - not all model parameter combination possible are realistic or observed in nature



Mai & Beroza (2000)

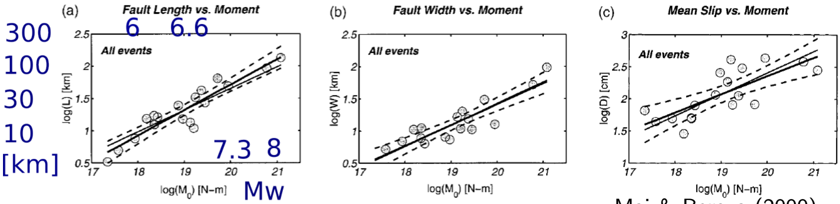
Magnitude and Seismic Moment

Moment Magnitude (Aki & Richards) $Mw = \frac{2}{3} log(M_0) - 6.03$

Seismic Moment [Nm] $M_0 = rigidity(30GPa) \cdot slip \cdot length \cdot width$

Source scaling for earthquakes

- not all model parameter combination possible are realistic or observed in nature



Mai & Beroza (2000)

Deviations in scaling relations for different faulting styles:

For strike-slip earthquakes and increasing moment:

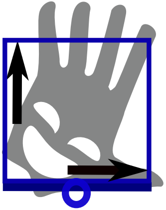
- the fault length grows faster than fault width

(fault width is even saturated)

- the fault area grows slower than for earthquakes on average
- the slip grows faster than for earthquakes on average

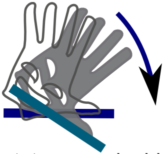
For dip-slip earthquakes and increasing moment the fault length and width grow similarly.

Disloc parameter conventions as right-hand law

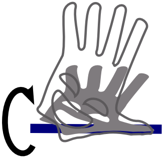


location (x,y,z)

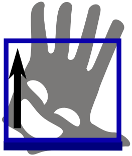
right hand is foot wall, the palm faut plane
(dip values 0 to -90 deg, negative only)
thump is fault's upper edge
thumb is pointing in strike direction
fingers point in dip-slip direction
thumb points in dextral strike slip



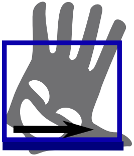
rotate around mid of
hand to rotate strike



rotate around thumb
axis to rotate in dip

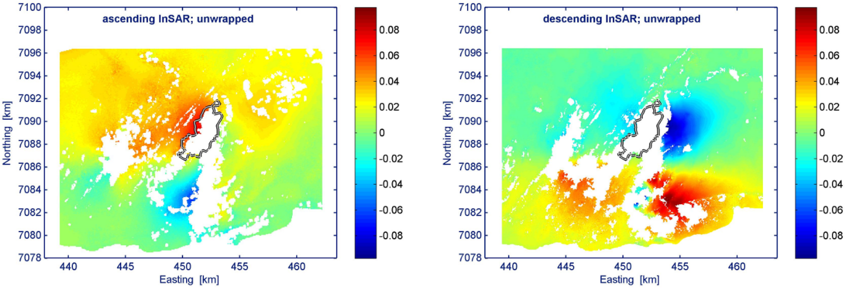


positive dip-slip: normal faulting
negative dip-slip: thrust fautling



positive strike-slip: right-lateral slip
negative strike-slip: left-lateral slip

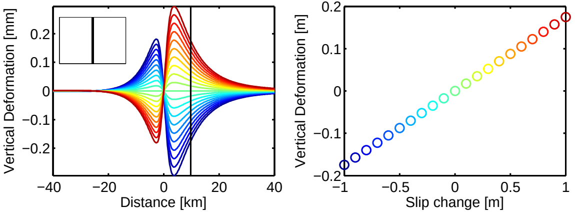
Case at hand: The 2000 Kleifarvatn earthquake



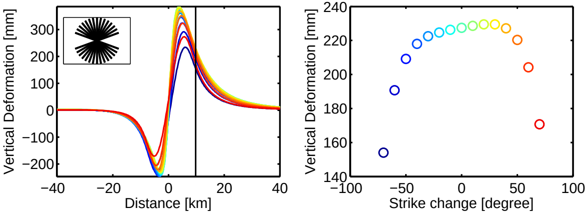
Sudhaus & Jonsson, 2009

The non-linearity in the dislocation modelling

Linear dependency:
Surface displacements for a vertical plane with varying slip



Non-linear dependency:
Surface displacements for a vertical plane with varying strike.



The non-linear inverse problem solved with “Direct Search”

The objective function

misfit (or ”cost“) function as a special objective function

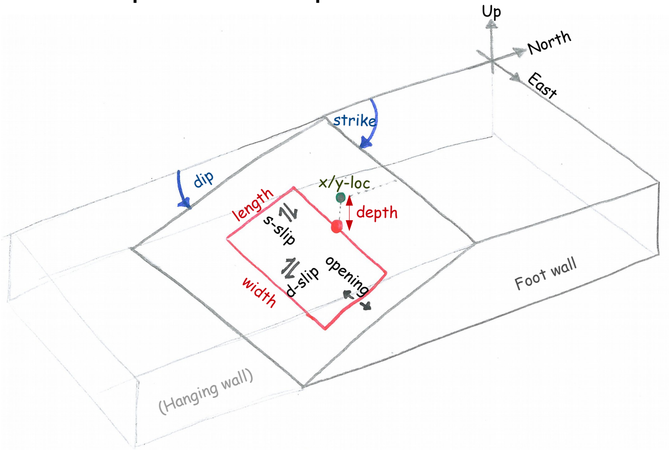
L_2-norm: misfit $\Phi(\mathbf{m}) = \|\mathbf{d}_{obs} - \mathbf{d}_{synth}\|_2 = \sqrt{(\mathbf{d}_{obs} - \mathbf{d}_{synth})^2}$
... very sensitive. Best suiting for highly non-linear problems.

The misfit function $\Phi(\mathbf{m})$ is a function of $\mathbf{m}$ since $\mathbf{d}_{synth} = \mathbf{G}(\mathbf{m})$.

$\Phi(\mathbf{m}) = \|\mathbf{d}_{obs} - \mathbf{d}_{synth}\|_2 = \sqrt{(\mathbf{d}_{obs} - \mathbf{d}_{synth})^2} = \sqrt{\mathbf{r}^{\mathrm{T}}\mathbf{r}}$, with $\mathbf{r}$ the residual.
Considered the data are noisy and/or are correlated we can apply a weighting in the misfit function to account for that:

$\Phi(\mathbf{m}) = \sqrt{(\mathbf{W}\mathbf{r})^{\mathrm{T}}(\mathbf{W}\mathbf{r})}$, with $\mathbf{W}$ being a of weighting vector, or
$\Phi(\mathbf{m}) = \sqrt{\mathbf{r}^{\mathrm{T}}\mathbf{\Sigma}^{-1}\mathbf{r}}$, with $\mathbf{\Sigma}$ being the data error covariance matrix.

The model parameter space



The Model Parameter Space

- is spanned by all (N) model parameters and is therefore N-dimensional.
- it is finite, however, given physical constraints and prior information.

Exercises - Direct Search #2 (automatically)

Purpose: Implementations and trying-out of simple Monte Carlo Optimizations.

On your virtual machines:
Go to **/home/jissy/Documents/2_Tuesday/EQ_geodesy/optitools_octave**
Open Nonlin_Kleifar_course_1.m

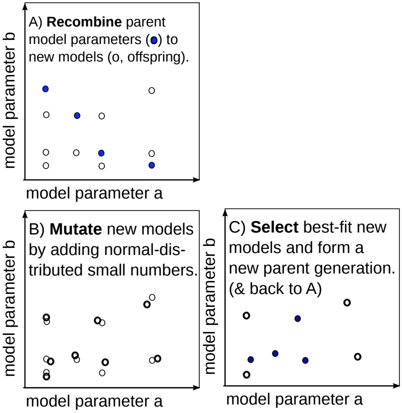
Editing:
Setting model parameter bounds: edit lines 99 & 100
Setting optimization options for evolution: edit line 148
run Nonlin_Kleifar_course_1 in octave
run Nonlin_Kleifar_course_2 in octave

or (for Simulated Annealing)
Setting optimization options: uncomment line 156
in **Nonlin_Kleifar_course_2.m:** comment line 9, uncomment line 11

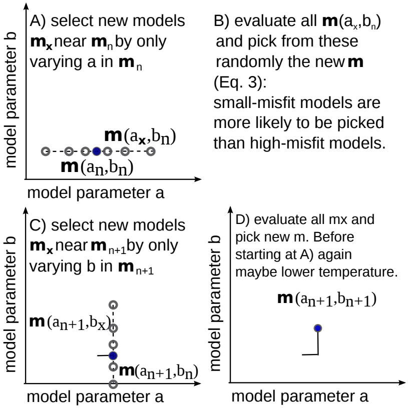
run Plot_Model_Parameters.m and Plot_Model_Predictions.m to see your results.

Exercises - Two Monte Carlo Flow Charts

Flow of an Evolutionary Algorithm



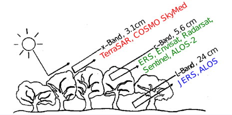
Flow of Simulated Annealing



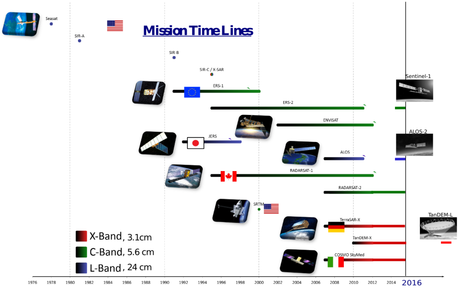
Extra Information on the following slides ...

Satellite missions: Time Lines and Specifications

SAR Roundtable, January 8, 2014



Mission Time Lines



Mission Specifications

name/ operated by	ERS-1/2, Envisat, Sentinel European Space Agency (ESA)	TerraSAR-X, TanDEM-X Deutsches Zentrum f. Luft- u. Raumfahrt (DLR)	JERS & ALOS-1/2 Japan Aerospace Exploration Agency (JAXA)	Radarsat-1/2 Canadian space agency (CSA)
online catalog	Catalog EOLI-SA (download and install from http://earth.esa.int/EOLi/EOLi.html)	EOWEB: https://centaurus.caf.dlr.de:8443/ infos: http://terrasar-x.dlr.de/	JERS: Catalog at https://www.gportal.jaxa.jp ALOS-1:Catalog: https://auig.eoc.jaxa.jp/	https://neodf.nrcan.gc.ca/neodf_cat3/
revisit times	ERS-1/2, Envisat: 36 days (future) Sentinel-1: 12 days	Terra-SAR-X & Tandem-X: 11 days (future) Tandem-L: no info found	JERS & ALOS-1: 36 days (future) ALOS-2: 14 days	Radarsat-1/2: 24 days (future) Radarsat Constellation: 4 days!
Band	C-band	X-band	L-band	C-band
coverage	Huge data set with world wide coverage	Coverage only on demand – no background mission!	Global coverage with a concentration in Asia	Focus on permafrost/ice covered globe, good coverage over northern hemisphere and science hot spots, e.g. Iceland, Hawaii
data availability (for research)	More or less freely available for registered users via EOLI-SA Sentinal data @ https://scihub.esa.int	Research scene charges are about 400 EUR, for researchers with a proposal at DLR a limited amount of data is free - check for opportunities (in particular for GFZ access is easier, in case ask T. Walter or M. Motagh).	JERS: normal charges ~19 US $ per scene, ALOS: Access via EoLiSA for some (!) areas Or (JERS & ALOS) for free for researchers with an accepted proposal (check for opportunities* at http://www.eorc.jaxa.jp/ALOS/en/ra/schedule.htm	Available for paying costumers at a charge of 3000 - 4000 EUR per scene, or some few scenes (<20) via research proposals (check for opportunities* at http://www.asc-csa.gc.ca/eng/ao/default.asp)
varia	Future mission Sentinel-1: 2014 - ? Ground resolution: 5 m in az / 25 m in range (less for Envisat Wideswath data) Swath width: 100 km (200 km for Envisat wideswath data) Extras: No GPS on board, therefore extra orbit data are needed from Delft: http://www.deos.tudelft.nl/ers/precorbs/orbits/ ERS-2 had from Feb 2001 troubles with stabilizing the sensor's orientation, resulting in big Doppler variations. Some scene pairs may therefore not work for interferometry	Swath width is different for different observation modes (2 km up to 200 km) Extras: there may be restrictions to order/get data over military-wise sensitive areas involving a lot of administration and also money (at least 50 per scene or 200 EUR)	Ground resolution 4 m in azimuth, 25 m in range, Swath with ~70 km	*short portraits* COSMO Skymed, by Italien Space Agency (AZI), is a constellation of 4 X-band SAR satellites, short revisit times, info at http://www.e-geos.it, operating since 2007 with two sat's, restricted data availability. RISAT-1, by the Indian Space Research Organisation (ISRO), operating since April 2012, C-band satellite.

GPS Data

Data Sources:

- from collaborators or your own network
- from published work
- https://unavco.org/data/data.html

Processing software:

- e.g. **GAMIT**, consult unavco page above
  Bernese: www.bernese.unibe.ch/
- https://unavco.org/data/data.html

InSAR Processing software

- **GMTSAR**: InSAR processing system based on GMT (Scripps/San Diego, Hawaii)
- **DORIS**: Delft object-oriented radar interferometric software (TU Delft)
- **ROI_PAC**: Similar to GAMMA, deprecated
- **ISCE**: InSAR Scientific Computing Environment (JPL, Caltech, Standford), ROI_PAC offspring

GAMMA: commercial, GAMMA Remote Sensing AG, Switzerland

SARscape: commercial, but the only software with a graphical user interface, based on ENVI, SARmap SA, Switzerland

...

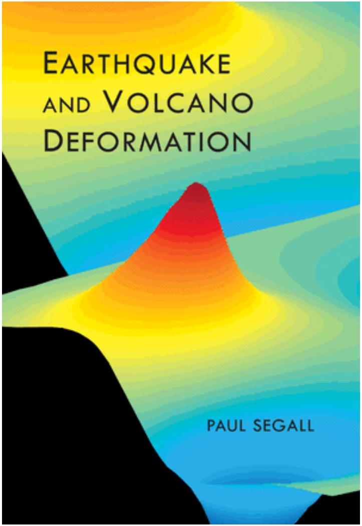
InSAR Time-series solver

- **StaMPS**: Stanford Method of Persistent Scatterers, developed by Andy Hooper (Stanford U, TU Delft, Leeds U), based on DORIS output, PS technique
- π-**RATE**: (Poly-Interferogram Rate And Time-series Estimator), Matlab toolbox developed by Biggs, Elliott, Wang (Leeds U), uses full interferograms processed with ROI_PAC
- **GIAnT**: Generic InSAR Analysis Toolbox, based on ISCE

  IPTA (GAMMA): Interferogram Point Target Analysis, PS technique

...

Literature on deformation modeling



Chapters:

- Deformation, Stress, Conservation Laws
- Dislocation Models of Strike-slip Faults
- Dip-Slip Faults and 3D Dislocations
- Crack Models of Faults
- Elastic Heterogeneity
- Postseismic Relaxation
- Volcano Deformation
- Topography and Earth Curvature
- Gravitational Effects
- Poroelastic Effects
- Fault Friction
- Interseismic Deformation

(library at Princeton Press, ∼90 EUR)

Literature on deformation modeling

On moodle:

- Okada, Y. (1985), Surface deformation due to shear and tensile faults in a half-space, *Bull. Seism. Soc. Am.*, 75(4), 1135-1154.
- Mogi, K. (1958), Relations between the Eruptions of Various Volcanoes and the Deformations of the Ground Surfaces around them, *Bull. Earthquake Res. Inst.*, 36, 99-134.