Joint inversion of first arrival travel times and gravity data

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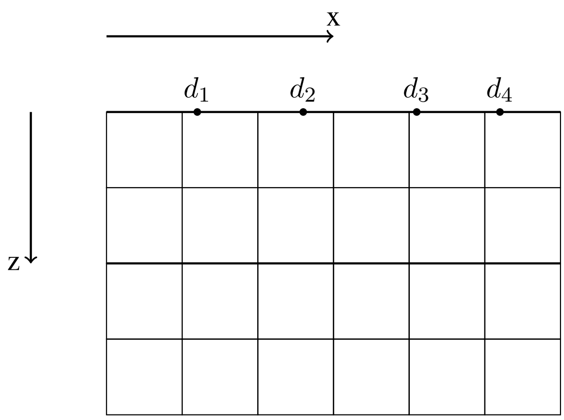
In this practical we will look at jointly inverting first arrival travel-time data together with gravity data. We will explore different types of coupling these different types of data and what implications this has on the inversion results. We will use the code described in Moorkamp et al. [2011] and the examples are similar to the ones shown in this paper. A practical application using this code can be found in Moorkamp et al. [2013].

General setup

First arrival travel times are sensitive to the P-wave velocity structure, v_p within the Earth, while gravity data are sensitive to density, ρ. We divide the Earth into a rectilinear mesh with constant velocity/density in each cell as shown in the figure below for two dimensions.



If we give each cell a number, we can describe the velocities/densities in this grid by vectors

$$\boldsymbol{\rho} = \left(\rho^1, \ldots, \rho^N\right) \qquad \boldsymbol{v_p} = \left(v_p^1, \ldots, v_p^N\right) \tag{1}$$

where $N = N_x \cdot N_y \cdot N_z$ and N_x, N_y and N_z are the numbers of cells in x,y and z-direction, respectively. From these vectors we can then form the model vector $\mathbf{m}$ for the inversion. How we do this depends on the type of coupling that we are using (see below).

Cross-gradient coupling

The cross-gradient [Gallardo and Meju, 2003] is a popular criterion to couple different physical properties in a joint inversion

$$\Phi_{cross}\left(\rho^i, v_p^i\right) = \nabla\rho^i \times \nabla v_p^i = \begin{pmatrix} \frac{\partial \rho^i}{\partial x} \\ \frac{\partial \rho^i}{\partial y} \\ \frac{\partial \rho^i}{\partial z} \end{pmatrix} \times \begin{pmatrix} \frac{\partial v_p^i}{\partial x} \\ \frac{\partial v_p^i}{\partial y} \\ \frac{\partial v_p^i}{\partial z} \end{pmatrix}. \tag{2}$$

Here, $\nabla\rho^i$ is the spatial derivative of density in the ith cell. Typically the cross-gradient is added as an additional term to minimize in the inversion, i.e. we are looking for models with cross-gradient values close to zero. For this approach the model vector consists of both the velocity vector and the density vector

$$\mathbf{m} = \left(\rho^1, \ldots, \rho^N, v_p^1, \ldots, v_p^N\right).$$

Questions

1. Under which circumstances does the cross-gradient become zero in a given cell?
2. Which geological situations does this correspond to?
3. When will the cross-gradient be significantly different from zero and what geological situation could that be?

Coupling through parameter relationships

Another way to couple velocity and density is through an empirical parameter relationship. There are many different possible form, we use

$$\rho = \frac{v_p + b}{a} \text{ with } a = 5m^4/(kgs) \qquad b = 8500m/s \tag{3}$$

In this case the model vector only consists of the velocities as we can uniquely determine density from the velocity values

$$\mathbf{m} = \left(v_p^1, \ldots, v_p^N\right). \tag{4}$$

Questions

1. Sketch the relationship for typical values of v_p within the Earth. Are the resulting densities realistic for typical Earth materials?
2. What happens under this assumption when seismic velocity changes within the Earth?
3. In which situations is such a change realistic?
4. What are situations where such a change is known to be wrong?

Joint inversion experiments – Coincident structures

The following experiments assume some familiarity with Linux and shell scripts. Open a terminal window in the Linux virtual machine (black screen symbol in the bar at the bottom) and change to the directory `~/Desktop/jif3D/Examples/Example1`. Run the script `makemodels`, this generates a test model, calculates synthetic travel times and gravity data and generates grids that we can use as starting models for our inversion. All data and model files are in NetCDF format (`http://www.unidata.ucar.edu/software/netcdf/`) a versatile storage format for scientific data. We are not going to look at the structure of these files in any detail, but it is easy to read/write them from Matlab, Python and most programming languages. Also, the .grd files produced by gmt are in nectdf format.

The directory `Example1` contains a bash script `run_rel`, which should look something like this

```
#!/bin/bash
jointinv  --writerays --substart True --curvreg --scalrelerr 0
     --scalminerr 5e-7 --ftgminerr 1e-9 --pickerr 0.02 << eof
start_tomo.nc #geometry of the inversion grid
1  # weight for tomography data
start_tomo.nc # starting model for slowness
tomo_ano.nc.tt.nc # travel time observations
```

```
1 # weight for scalar data
1 #weight for tensor data
dens_ano.nc.sgd.nc # scalar gravity observations
dens_ano.nc.ftg.nc # tensor gravity observations
dens_ano.nc # starting model for density (for background model)
0 # weight for magnetotelluric data
100 # weight for regularization
100 # maximum number of iterations
eof
```

This script runs a joint inversion of travel-time data together with scalar and tensor gravity data. The program uses a mix of command line parameters and asks for the essential information (the bits between `<< eof` and `eof`). If you just start the executable, you will see a prompt where you can also type in the information.

Tasks

1. Run the script as it is. It will produce various files with names starting with `result` and some summary information in `misfit.out` and `rms.out`.
2. Look at the file `rms.out`. It shows the evolution of the root mean square misfit

$$\text{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(d_{obs}-d_{synth})^2}{\sigma^2}}$$

   with iteration. What do these values tell us?
3. Open the inversion models in `paraview` (I will show you how) and look at the evolution of the models. What do you see?
4. Run individual inversions for the different datasets. You can do this by setting the corresponding weights to very small values (This way you will also get the hypothetical misfit for the other dataset). Compare with the joint inversion results.
5. Vary the weights for the different datasets and the regularization. What impact does this have on the final models and the data fit? Note down the final data fit for different weights of the data and plot those values for tomography vs. one of the gravity datasets. What do you observe?
6. You can use two command line parameters `--density_a` and `--density_b` to change the parameters in the relationship (equation 3) from the default values. At the moment they coincide with the values used to generate the synthetic data. What happens when you change one or both of them? How can you understand if you are using the right values?

The script `run_cross` shows how to run a joint inversion using cross-gradient coupling:

```
#!/bin/bash
jointinv --crossgrad --writerays --substart True --curvreg
        --scalrelerr 0
      --scalminerr 5e-7 --ftgminerr 1e-9 --pickerr 0.02 << eof
start_tomo.nc #geometry of the inversion grid
1  # weight for tomography data
start_tomo.nc # starting model for slowness
tomo_ano.nc.tt.nc # travel time observations
1 # weight for scalar data
1 #weight for tensor data
dens_ano.nc.sgd.nc # scalar gravity observations
dens_ano.nc.ftg.nc # tensor gravity observations
dens_ano.nc # starting model for density (for background model)
```

```
0 # weight for magnetotelluric data
0 # weight for slowness−conductivity CG (not used)
1e5 # weight for slowness−density CG
# weight for density−conductivity CG (not used)
100 # weight for slowness regularization
100 # weight for density regularization
0 # weight for conductivity regularization (not used)
100 # maximum number of iterations
eof
```

We now have to specify the weights for the cross-gradient terms and we regularize each model parameter separately.

Tasks

1. Repeat the experiments described for the parameter coupling above for the cross-gradient.
2. Also, vary the weight for the cross gradient constraint (by factors of 10). What impact does this have on the inversion?

Joint inversion experiments – Differing structures

The directory `Example2` contains an example where the density structure differs from the velocity structure in a part of the model. Repeat the experiments above for this example.

References

L. A. Gallardo and M. A. Meju. Characterization of heterogeneous near-surface materials by joint 2D inversion of dc resistivity and seismic data. *Geophysical Research Letters*, 30(13):1658, 2003.

M. Moorkamp, B. Heincke, M. Jegen, A. W. Roberts, and R. W. Hobbs. A framework for 3-D joint inversion of MT, gravity and seismic refraction data. *Geophysical Journal International*, 184:477–493, 2011. doi: 10.1111/j.1365-246X.2010.04856.x.

M. Moorkamp, A. W. Roberts, M. Jegen, B. Heincke, and R. W. Hobbs. Verification of velocity-resistivity relationships derived from structural joint inversion with borehole data. *Geophysical Research Letters*, 40(14):3596–3601, 2013. ISSN 1944-8007. doi: 10.1002/grl.50696. URL `http://dx.doi.org/10.1002/grl.50696`.